TESTING TWO EXISTING FERTILIZER RECOMMENDATION ALGORITHMS: STANFORD'S 1.2 RULE FOR CORN AND SITE-SPECIFIC NUTRIENT MANAGEMENT FOR IRRIGATED RICE

BY

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DISSERTATION

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ABSTRACT

This dissertation evaluates two existing fertilizer recommendation algorithms that are based on yield goal approach: Stanford's 1.2 Rule for corn in the United States and site-specific nutrient management (SSNM) for rice, which uses an algorithm similar to the 1.2 Rule, in Southeast Asia. Fertilizer recommendations all over the world have relied heavily on an old but widely accepted rule of thumb from Stanford (1966, 1973): *apply 1.2 pounds of nitrogen (N) fertilizer per bushel of corn expected.* While algorithms similar to the "1.2 Rule" have been used for the past four decades all over the world to make fertilizer recommendations for various crops, little is known about the 1.2 Rule's origin. I use microeconomic analysis to examine the historical origins of the 1.2 Rule and show that the 1.2 Rule only makes economic sense if the crop production satisfies two restrictions: (1) it is of the von Liebig functional form, i.e. the function has a "kink" and a "plateau," and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with slope 1.2. To investigate if the 1.2 Rule satisfies these restrictions, I utilize the original dataset Stanford used in his analysis and the long-term experimental data on corn from Illinois, Iowa, and Nebraska. I find no empirical evidence to support the 1.2 Rule using non-linear estimation techniques and a non-nested hypothesis framework. The crop production function and the critical concentration of *N* vary across and even within fields, and hence site-specificity matters in making fertilizer recommendations.

I also critically discuss and evaluate the assumptions underlying the SSNM strategy for rice in the top rice producing countries in the world: India, Indonesia, Philippines, Thailand, and Vietnam. I find clear evidence that interaction among major nutrients matters in making fertilizer recommendations to farmers. The relationships among *N*, *P*, and *K* vary across sites -- *some* *inputs are complements, some are substitutes, and some are independent*. I also find that soil organic matter, manifested in soil *C* stocks, significantly affect the economic returns to *N* fertilizer inputs. The marginal product on *N* is low on soils with low *C* content. These results suggest the SSNM strategy should explicitly account for the: (1) nutrient interactions and (2) relationship of *N* fertilizer and soil organic matter, as reflected in soil *C* stocks. In addition, input and output prices should also not be ignored in SSNM algorithm. The major challenge for SSNM strategy will be to retain the simplicity of the approach that is understandable to producers and extension agents while accounting for the relationship of *NPK*, soil organic matter, and prices. Clearly, this is an area of research in great need of interdisciplinary collaboration among agronomists and agricultural economists.

To my parents, Mario and Aida

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LIST OF ACRONYMS

AD	Aduthurai, India
AL	Alabama
Bb	Box Butte
Be	Belle Mina
Bk	Brunswick
Br	Brosius
Bv	Brooksville
Bw	Bellwood
C	Carbon
Ca	Cairo
CC	Clay Center
Co	Concord
CT	Can Tho, Vietnam
EONR	Economically optimal nitrogen rate
Fk	Funk
GA	Georgia
HA	Ha Noi, Vietnam
HYS	High yielding season
IA	Iowa
IDR	Indonesian Rupiah
IL	Illinois
INR	Indian Rupee

IRRI	International Rice Research Institute
K	Potassium or Potassium fertilizer applied
Md	Mead
Mo	Monmouth
MPP	Marginal physical product
MRTN	Maximum return to nitrogen
MS	Mississippi
MVP	Marginal value product
N/N_F	Nitrogen or Nitrogen fertilizer applied
NB	North Bend
NE	Nebraska
NJ	Nueva Ecija, Philippines
NP	North Platte
NPK	Nitrogen, Phosphorus, and Potassium
Or	Orr
N^{up}	Nitrogen uptake
P	Phosphorus or Phosphorus fertilizer applied
Pa	Paxton
PhP	Philippine Peso
Pk	Pickrell
Po	Poplarville
Pr	Pratville
SB	Suphan Buri, Thailand

SC	SCAL
Sf	Scottsbluff
Sp	Spurgin
SSNM	Site-specific nutrient management
SU	Sukamandi, Indonesia
Th	Thorsby
Ti	Tifton
TJ	Thanjavur, India
UP	Uttar Pradesh, India
VND	Vietnam Dong
Wa	Watskinville
Wy	Wymore
Y^{DM}	Dry matter yield
$\bar{Y}^{DM}$	Total dry matter yield

Chapter 1

INTRODUCTION

The role of fertilizers in agricultural production, particularly for corn and rice, cannot be understated. Intensification of agriculture through the use of fertilizer remains one of the most likely options for increasing agricultural productivity in many parts of the world. Some argue that fertilizer was as important as seed in the Green Revolution (Tomich et al., 1995). Chemical fertilizer is responsible for 40 to 60 percent of the world's food production (Erisman et al., 2008). The worldwide consumption of nitrogen-phosphorus-potassium (*NPK*) fertilizer in agriculture increased from 144 million tons in 2002 to 182 million tons in 2010 (Figure 1.1.), corresponding to 25% increase *NPK* use. Specifically, *N* fertilizer consumption from that 8 year-period (2002-2010) increased from 86 to 112 million tons – a 30% increase in *N* fertilizer total usage. Rice production alone uses 16 percent of total *N* fertilizer and 15 percent of all fertilizers worldwide (Heffer, 2008). Because fertilizer is such an important input, agricultural production must become more efficient in the use of fertilizer and essential plant nutrients. But in the pursuit of food security, the inappropriate application of fertilizer has been a common practice in many agricultural crop production systems all over the world. Globally, under- or over-fertilization of crops leads to either lost agricultural production or environmental degradation.

To ensure that *N* and other essential plant nutrients are applied optimally and are readily available during crop growth periods, it is critical to define and establish an appropriate fertilization rate, which is the foundation to science-based nutrient management (Chuan et al., 2013). Crop production needs appropriate fertilization strategies, a recurrent challenge for the farmer before and during each cropping period. Fertilization-recommendation algorithms must adequately account for nutrient interactions as the driving force behind plant uptake. Addressing this challenge requires knowledge of how crop yields respond to fertilizer and other factors that influence crop growth, and how those responses differ across and even within fields. Understanding the crop response function to fertilizer may facilitate the development of better means of forecasting how to adjust *N* fertilizer levels that raise farmers' profits. As Cassmann et al. (2002, p. 132) put it,

> *Achieving synchrony between N supply and crop demand without excess or deficiency is the key to optimizing trade-offs amongst yield, profit, and environmental protection in both large-scale systems in developed countries and small-scale systems in developing countries.*

This dissertation evaluates two existing fertilizer recommendation algorithms: Stanford's "1.2 Rule" for corn, and the site-specific nutrient management (SSNM) for irrigated rice systems, which also use an algorithm based on the 1.2 Rule. This dissertation critically discusses some of the assumptions underlying the 1.2 Rule and SSNM, as well as their potential for improving corn and rice production.

For the past several decades, fertilizer recommendations all over the world have relied heavily on a four-decades old but widely accepted rule of thumb from Stanford: *apply 1.2 pounds of N fertilizer per bushel of corn expected.* By positing that in all cornfields under all conditions the economically optimal *N* application rate equals a fixed proportion of yield potential, Stanford's 1.2 Rule seemingly obviated the need to know how yields actually respond to *N* (Raun, 2002). Stanford's 1.2 Rule (or the "yield-goal base approach" to *N* fertilizer recommendation) has been the basis upon which the vast majority of researchers, extension personnel, and policy makers have evaluated *N* practices over the past several decades.

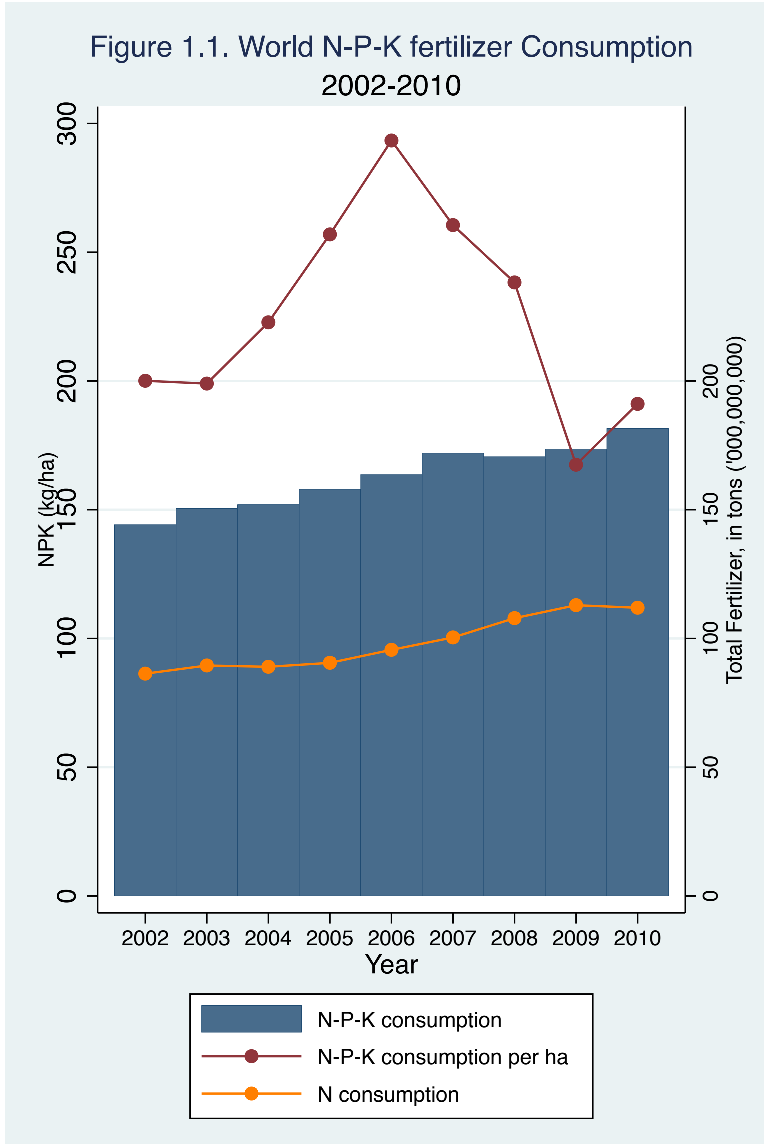
Numerous studies have pointed out problems in the 1.2 Rule. My dissertation does not simply critique and examine the historical origins of the 1.2 Rule, but also provides discussion of its origins and empirical analysis of its application, both to judge its appropriateness and to examine how better recommendations might be made.

In Chapter 2 of this dissertation, I discuss the research from which Stanford's 1.2 Rule was developed, how it was formulated, and the reasons for its particular formulation. Much of the recent literature on the yield goal approach casts doubt over its efficacy, but no one has investigated the Stanford's research designs and analyses. I use microeconomic analysis to show that Stanford's 1.2 Rule only makes economic sense if the crop yield response function satisfies two restrictions: (1) it is of the von Liebig functional form, i.e. has a "kink" and a "plateau," and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with slope 1.2.

In Chapter 3, I examine the original dataset Stanford used in his analysis and the long-term experimental data on corn from Illinois, Iowa, and Nebraska to investigate if the 1.2 Rule satisfied the restrictions mentioned above. I show that contemporary estimation techniques shed doubt on the validity of the Stanford's 1.2 Rule. The von Liebig crop response model is used as a starting point to test its appropriateness. Many researchers have assumed that response functions can be best estimated assuming polynomial functional forms. In this regard, I apply non-nested hypothesis tests using the polynomial response models as the rival hypotheses to the von Liebig response model, to determine the correct crop-nutrient model. As I will explain later in the text, this issue is critical, as the shape of the yield response function has significant economic implications: if the production function is von Liebig, then input and output prices will not affect the economically optimal *N* fertilization rate.

In Chapter 4, I critically discuss and evaluate the assumptions underlying the SSNM strategy for rice, which uses an algorithm similar to the 1.2 Rule, in the top rice producing countries in the world: India, Indonesia, Philippines, Thailand, and Vietnam. Like Stanford's 1.2 Rule, the total fertilizer needed by rice to achieve a profitable target yield is determined from the anticipated yield gain ("yield potential"), applied fertilizer and nutrient supplied by the soil, and a targeted efficiency of fertilizer use. I emphasize an underlying assumption about the relationship between major nutrients and soil organic matter, and the assumption's implications for fertilizer recommendation. I explore whether major nutrient inputs are substitutes or complements, and if there are complementarities between inorganic fertilizer and soil organic matter. These issues are critical in the decision-making process of policymakers from the above-mentioned countries, and the path that these countries choose to take with fertilizer policy has significant implications for food security through the global market for rice.

In Chapter 5, I summarize the results and the overall policy implications of the dissertation. Important lessons can be learned from my study about the process in which university- and research institution-led farm management guidelines are developed.



Source: FAOSTAT, 2012

Chapter 2

AN EXAMINATION OF THE STANFORD "1.2 RULE" FOR NITROGEN FERTILIZER

Stanford's (1973) yield-goal based algorithm, known as the **"The 1.2 Rule"** (Halbeisen, 2006), has been widely used over the past four decades by university extension and private consultants to recommend nitrogen (*N*) fertilizer rates to farmers. Similar rules have been established throughout the world for other crops, including soybeans in Canada (Janovizek 2011), and rice in the Philippines (Witt et al., 1992). The 1.2 Rule instructs the farmer to begin with a number which represents a type of per-acre yield on the field to be fertilized. Often this yield is the "yield goal" or "target yield," and is described as representing the yield that the farmer "hopes to achieve" on his field (Illinois Agronomy Handbook, 1999-2000). Often, it is interpreted as the "potential yield," that is, the highest that the farmer believes could be gotten on the field (Dahnke et al., 1992). The 1.2 Rule is that for every bushel of corn a farmer wishes to grow (yield goal or target yield) or thinks is possible to be grown (yield potential) on his field he should apply 1.2 lb of nitrogen fertilizer per acre, with adjustments for previous crops grown and other factors. Nafziger, et al. (1998, p. 89) wrote "[t]arget *yield is one of the major considerations in determining the optimum rate of nitrogen application for corn."*

Many fertilizer N recommendations for U.S. corn grain take the following form:

$$N_f\,(\text{lb acre}^{-1}) = 1.2\ YG\,(\text{bu acre}^{-1}) \ \ - N_s \qquad (2\text{-}1a)$$

where N_f is the estimated economically optimum N rate, N_s is the quantity of N supplied by the soil and YG is the yield goal or the target yield. Lory and Scharf (2003) report that the state university systems following or having followed this yield goal approach include Illinois (Hoeft and Peck, 2001), Minnesota (Schmitt et al., 1998), Missouri, Nebraska (Hergert et al., 1995), North Dakota (Dahnke et al., 1992), Pennsylvania (Beegle and Wolf, 2000), and South Dakota (Gerwig and Gelderman, 1996). Indiana, Michigan, and Ohio (tri-state) follow the same approach but use the following formula (Vitosh et al., 1995):

$$N_f\,(\text{lb acre}^{-1}) = 1.36\ YG\,(\text{bu acre}^{-1}) - N_s\,(\text{lb acre}^{-1}) - 27 \qquad \text{(2-1b)}$$

Wisconsin (Bock and Hergert, 1991) uses the form:

$$N_f\,(\text{lb acre}^{-1}) = \frac{1.13YG}{E_f}(\text{bu acre}^{-1}) - N_{OM}\,(\text{lb acre}^{-1}) - N_r\,(\text{lb acre}^{-1}) \qquad \text{(2-1c),}$$

where E_f is the fractional recovery of N_f by the crop, N_{OM} is the net N produced from mineralization of organic matter (OM), and N_r is the amount of available residual mineral N in the root zone. Other examples of current corn N fertilizer recommendations based on this rule of thumb are presented in Table 2.1, where the fertilizer N recommendations are in the 1.0 to 1.2 lb-N/bu range.

While algorithms similar to the 1.2 Rule have been used for decades all over the world to make fertilizer application rate recommendations for various crops, remarkably little is known about its origin. Stanford (1973) is rarely cited when the 1.2 Rule is invoked. With its origin lost, to a good extent the perceived legitimacy of the Rule results simply from its long-time and widespread use, not its scholarly origin or demonstrated scientific legitimacy. In this section, I discuss the research from which this 1.2 Rule was developed, how it was formulated, and what were the reasons for its particular formulation. I also use basic microeconomic analysis to critique the 1.2 Rule. Most importantly, I show how Stanford's lack of access to modern statistical analysis and to microeconomic principles shed doubt on the validity of his 1.2 Rule. I conclude that use of the 1.2 Rule is the result of long-term and widespread failure of university research and extension.

2.1. Stanford's Formula – the Origin of the 1.2 Rule

2.1.1. Stanford's motivation: "a less empirical approach" for farmers is needed

Stanford (1966) attempted to develop improved procedures for estimating optimum nitrogen fertilizer. Stanford (1966, p. 237) commented,

> [i]*n formulating recommendations for nitrogen fertilizer use, agronomists and soil scientists have relied mainly on experience and interpretations of the numerous field and associated laboratory studies conducted over the years. These efforts have served the farmer and the agricultural chemical industry well. Future progress, however, demands that* ***less empirical means be developed for predicting and meeting the nitrogen needs of crops*** [bold typeface added].

As proposed by Heady and Pesek (1954), large agronomic experiments[1] and replications must be performed to better understand the form of the yield response to inputs, and to account for site differences. Given the technology available to experimenters in the 1950s, running agronomic experiments in every farmer's fields to estimate optimal *N* rates was clearly infeasible. While farmers have a good understanding of the importance of *N* in production, they rarely have detailed knowledge of the empirical relationship between yield and *N* on their specific fields. Therefore, Stanford's purpose was to present an alternative approach to fertilizer

[1] Here "large" means "plots are small, but numerous."

recommendation that a farmer could use without having to perform agronomic experiments. Stanford's (1973, p. 160) objective was

> *to develop a rational basis for maintaining the levels of N fertilizer use within bounds that not only are optimum for crop production, but also provide for an acceptable balance between N inputs and losses of nitrate to surface and ground waters.*

To provide *N*-rate recommendations to farmers, there was a need for a "rule of thumb" to estimate optimal fertilizer management of crops when knowledge of yield response functions and access to experimental data are not available. Using Truog's approach (as we will discuss later) and assuming that *N* taken up from the soil and fertilizer are recovered by the plant with equal efficiency, Stanford (1973) defined the *N* fertilizer requirement as the difference in *N* uptake by plants receiving fertilizer *N* and plants receiving no fertilizer, divided by the fraction of the *N* fertilizer recovered by the crop. The above-ground *N* contained in a crop with a specified yield (*N* uptake), and the amount of *N* supplied by the soil to the above ground portion of the crop (*N* supply) are considered to predict the fertilizer *N* requirements. The difference between the two *N* is implicitly the deficit of N in the system. The aim of Stanford's 1.2 Rule is to compensate for this deficit.

2.1.2. Historical Intellectual Background for Stanford's Thoughts and Methods

To help farmers make better decisions on fertilizer input and output levels in crop production, agronomists made great efforts to appropriately estimate crop response functions. These agronomic thoughts have had substantial impacts on the development of the 1.2 Rule. This section discusses a brief overview of the history of early ideas in agronomy that influenced Stanford's 1.2 Rule.

2.1.2.1. Origin: von Liebig's theory of plant response to inputs

The theory of crop response started with von Liebig's **'Law of Limiting Factors'** or **'Law of the Minimum',** which he expressed in relation to fertilizer use on crops. In 1855, von Liebig (p. 223) stated:

> *Every field contains a maximum of one or more and a minimum of one or more different nutrients. With this minimum, be it lime or any other nutrient, the yield of crops stands in direct relation. It is the factor that governs and controls the amount or duration of the yields. Should this be minimum for example lime …, the yield … will remain the same and be no greater even though the amount of potash, silica, phosphoric acid, etc., … be increased a hundred fold.*

Many authors have interpreted this as an assumption of Leontief technology.

> *It* [Law of the minimum] *ended up embraced by economists almost one hundred years later in a more rigid specification known as the Leontif model, which has been widely applied as a research and policy tool.* (Grimm, Paris, and Williams 1987, p. 191)

The Law of the Minimum suggests that plant growth is limited by a single resource at any one time and yield is directly proportional to the quantities of that limiting nutrient available in the soil (whether naturally soil-borne or in the soil due to fertilization). The implications of Liebig's statement is that the isoquants of the crop production function, as shown in figure 2.1a, have vertical and horizontal legs that join at right angles, and that the Liebig production function when charted as dependent on nitrogen (with other inputs held constant) is kinked, as shown in figure 2.1b.[2] This law clearly tells us that if a soil is deficient in, say, nitrogen, yields will be unresponsive to increases in phosphorus and potassium, as is the case for a Leontief production function. The term "direct proportion" used by von Liebig is widely interpreted as implying the "linear response and plateau" model (Paris, 1992); under this interpretation, the law of the minimum implies that if the nutrient is limiting, its marginal product is constant, and if the nutrient is no longer limiting, its marginal product is zero. This supposes that a given level of yield can be attained only by use of a single combination of inputs. Any change in ratio of input prices does not affect the fixed proportion in which inputs are combined in the production process. Any price ratio between the inputs will always go through where the kink is. The two elements are "technical complements" and if they are to be used at all, they should be used in this single combination.

As Bray (1954, p.9), noted, Liebig's law of the minimum could be interpreted to mean that the crop used up all the deficient nutrient in the soil, making yield directly proportional to the amount of deficient nutrient present and the crop content of that nutrient. In fact, the prevalent viewpoint is that von Liebig hypothesis implies a Liebig's statement also conveys the twin notions of non-substitution among nutrients and of yield plateau (Paris, 1992). The zero input elasticity of substitution ($\sigma = 0$) implies that cost-minimizing input quantities are independent of input and output prices, which therefore can be ignored in making fertilizer recommendations. This is not the case usually assumed or analyzed by economists, who tend to think that prices do matter. Yet, Liebig's ideas have dominated the thinking of agricultural

[2] An isoquant is a locus of points (curve or line) representing the various combinations of two inputs that can be combined to produce the same output.

scientists and have been of universal importance in soil fertility management. Even economists Heady and Dillon (1961, p. 10), who spent their careers estimating differentiable production functions, wrote,

… most production functions probably have a von Liebig point.

2.1.2.2. The Law of Diminishing Marginal Productivity

The Law of Diminishing Marginal Returns states that, if either N fertilizer or K fertilizer is increased alone, or if both are increased in constant proportions, this will increase the amount of corn yield. However, as you keep increasing *N* use or *K* use or both, its effectiveness will eventually diminish—that is, its marginal product will fall.

The production function that exhibits diminishing marginal product is represented by figure 2.2 (upper panel). The diminishing marginal products are expressed when each succeeding increment of fertilizer causes a smaller increase in yield than the previous addition of fertilizer. Instead of the assumed linear relationships in Liebig's law, the law of diminishing marginal product is based on observations of curvilinear relationships. In Stage 1, it increasing a variable input with other inputs fixed increases output per unit of the varied input, the latter reaching a maximum point where the average product of *N* is at its maximum point. At Stage I, each additional unit of fertilizer contributes more to the production than the previous unit of added fertilizer. Since the output per unit of the variable input is improving throughout this stage, a farmer will always produce beyond this stage. In Stage II, output increases at a decreasing rate and the marginal product begins to decline (Figure 2.2, lower panel). Diminishing marginal returns of fertilizer occurs. In Stage III, addition of another unit of fertilizer will cause the total yield to decline. Note that the slope of the production function at any one point indicates the marginal product of fertilizer. As I will discuss later, the marginal product schedule and the input and output prices are important in the determination of the economic optimal fertilizer rate.

2.1.2.3. Precursors to the 1.2 Rule: Early Mass Balance Approaches, Implicitly Assuming Leontief Production Functions

Stanford's 1.2 Rule was based on the idea of Truog (1960) and perhaps was a response to Viets's (1965) concern about the difficulty in predicting the total *N* uptake of crops. Truog's and Viets's fertilizer prescription implicitly requires knowledge of how yield responds to different *N* levels and other inputs. This section is a lengthy discussion of the Truog's and Viets's research, which focused on the initial justifications of early yield goal approaches used in *N* fertilizer recommendations that led to the development of the 1.2 Rule.

2.1.2.3.1. Emil Truog's prescription for fertilizing corn

Fertilizer recommendations based on yield potential started with Emil Truog in 1960. For land with a 100-bushel yield potential[3], Truog's fertilizer prescription was,

> [A] *100-bushel corn crop (ears and stalks) contains approximately 150 lbs of N, 60 lbs P_2O_5, and 120 lbs of K_2O. These amounts must be obtained by the corn from the soil, manure and fertilizer applied, otherwise the 100-bushel corn yield is not possible.* (page 48).

Truog's quote above reflects the general thinking of crop scientists of his time: that a corn plant will absorb no more nutrients than needed to produce its maximum yield potential.[4] The optimal amount of nutrients to provide the plant then is just enough, but no more, to allow the plant to

[3] Note that Truog's prescription was often based not on grain yield, but on the total mass of the corn plant (grains and stalks). This will become important later in our discussion.
[4] The excess *N* is not metabolized by the corn plant into a functional or structural compound.

achieve its yield potential. The plant no longer metabolizes the *N* beyond the optimum amount needed. Note that there are no explicit economics in this recommendation. The crucial rates of *N*, *P*, and *K* were estimated from an "average response" of plant to fertilizer *N* applied. Truog (1960) assumed that crop nutrients were only taken from the soil, manure and fertilizer, while the rate of fertilizer needed depended on the difference between the internal nutrient requirements for an expected attainable crop mass (ears and stalks) and mineralized nutrients supplied by the soil during the cropping season. This approach was later defined as the *mass balance approach*. The mass balance approach involves determining how much *N* will be available in the soil for crop uptake, and how much will be removed with the crop given *N* credits and debits (Meisinger, 1984). During the 1960s, agronomists accepted that as a general rule, only 50 to 60 percent of fertilizer nitrogen applied is absorbed by crops (based on long-term field experiments summarized by Allison (1955)). It is also implied that the amount of *N* required follows Liebig's concept of crop response, as Truog (1960) states,

> *..., increasing attention was being given to the supply of minor nutrient elements in soil as a factor that may limit crop yields*. (p. 47).

Though Truog (1960) does not explicitly mention it in his work, our interpretation is that he assumed that the crop response function reflects a Leontief-type technology, for his recommendation algorithm only makes economic sense if the yield response function reflects Leontief and Liebig technologies. That is, only if there is no substitution between inputs, and the production function is linear and plateaus after a kink, do prices not matter.[5] The farmer-

[5] Prices do matter, but only in extreme cases. For example, in the case of *1-input* and *1-output*, if *the* $w/p>MP$, any profit-maximizing producer will choose to produce nothing and if the $w/p<MP$ then a profit-maximizing producer will want to produce output in the kink.

producer would always combine the inputs in fixed ratios no matter the price of the inputs. If nitrogen is deficient in soil, increasing phosphorus and potassium will not increase yield.

Truog also assumed that *N* limited yield and that yield was directly proportional to the quantities of limiting *N* available in the soil. Figure 2.1a shows a plant needing *N* and *K* nutrients in a fixed ratio. If *N* is limiting to match *K*, the plant will respond to an additional amount of *N* by increasing yield at constant rate until the amount of *N* matches the predetermined ratio in relation to the amount of *K*. The elasticity of substitution between these two nutrients is 0 ($\sigma = 0$). Although statistical and mathematical tests of the nature of crop response to *N* were already available during the time when Truog was developing his ideas and conducting his research, he might have had little training and access to these methods, and hence simplified his analysis by assuming that crop response function reflects a Leontief-type technology.

Since Truog was trying to come up with general recommendations, his fertilizer prescription only implied that there are only three factors that are relevant to a farmer's decision making in corn production: nutrients in the soil, manure, and fertilizer despite agronomists recognizing that fertilizer recommendations should be based on data and principles drawn from both agronomy and economics. Other factors of production (e.g. rainfall, temperature, slope, prices, etc.) were not mentioned. He did not consider that the interactions of other factors could significantly affect optimal *N* fertilizer recommendations. Moreover, his fertilizer prescription aimed to achieve a certain biomass production (silage and stalks) and grain yield, and not grain yield alone.

2.1.2.3.1. Viets's alternative approach to fertilizer recommendation

2.1.2.3.1.1. Viets's mass balance equation

Following Truog's prescription for fertilizing corn, Viets (1965) provided one of the first equations defining the nitrogen requirement of a crop based on the mass balance approach. Viets (1965) emphasized that existing soil nutrients must be accounted for:

> *Knowledge of* ***how much is needed*** *in the crop is, of course, only a partial answer to the question of* ***how much nutrient you need to put on the soil****, for the fertilizer efficiency on soil does differ enormously.* (p. 7, bold typeface added).

Subject to climatic and soil conditions, Viets (1965) expressed fertilizer *N* need of the crop as the difference between the total *N* uptake of the crop and the amount of nitrogen obtained from the soil itself, divided by the fertilizer efficiency:

$$N_F = \frac{N_K^u}{E_F} - N_s \qquad \text{(2-2a)}$$

where N_F is the amount of *N* fertilizer application rate and E_F is defined as the proportion of fertilizer *N* in the soil that is taken up by the crop. The E_F, as mentioned earlier, is usually 50 to 60 percent. N_s is the amount of *N* available in the soil, measured through soil testing and the N_K^u is the sum of available soil *N* and *N* fertilizer absorbed by the plants, how much *N* is present in the harvested dry matter of the corn plant (including stover and grain). Viets and Truog used total *N* uptake and dry mass yield when considering a plant's total need for *N*. They did not examine how plants responded to different fertilizer rates.

Figure 2.3 illustrates how equation (2-2a) works. The corn plant obtains the nutrient from two sources: N_S and N_F. When $N_F = 0$ while $N_S > 0$ that can be absorbed by corn, the yield would be at Y_{Ns}. If a positive amount of N_F is applied, crop yield increases. To achieve Y_{Max}, the amount of N needed is N_P. If the farmer knows N_K^u, then the total N that must be applied is

$$N_P = \frac{N_P^u}{E_F} \qquad \text{(2-2b).}$$

The required N_F is the difference between N_P - N_S. At N_P if the amount of N_F is further increased, the crop yield will not be affected holding other factors constant.

2.1.2.3.1.2. Problems in accurately predicting yield and N content of a crop

To use equation (4), only knowledge of the plant's total *N* uptake, the amount of *N* present in the soil, and the efficiency of fertilizer *N* are needed. Although soil tests can tell how much *N* there is in the soil in available forms, N_K^u still cannot be accurately predicted because *N* in the roots is seldom known and total yield of the crop is seldom predictable, being subject to climatic and cultural conditions and to other factors besides *N* supply (Viets, 1965). Here, Viets recognized that other factors also influence the *N* demand of a crop. Analyses of field experiments data are based on the premise that a functional relationship exists between the yield of plants and fertility levels. He argued that the slope of the yield curve is often difficult to define *"with statistical significance because of **variability in yields** and the **lack of sufficient number of points*** [page 506, bold typeface added]**.***"* It was impossible to carry out sufficient agronomic experiments to cover every eventuality. Because of this, Viets (1965) recognized that it was difficult to generalize a response of plants to *N* fertilizer because one would have to extrapolate from the results of the few experiments that had been conducted.

In response to this issue, to come up with a reliable relationship of plant's response to *N,* most agronomists tended to approximate the average *N*-response from limited data sets. Viets (1965) acknowledged that an easier (and perhaps more reliable) way to depict the crop response to *N* fertilizer was to use a variety of mathematical functions to fit the experimental data. Viets (1965) did not emphasize any production function but suggested that functional relationship between the yield of a crop and nitrogen should conform to the principle of **diminishing marginal returns**. He added further,

> *There is no a priori basis for knowing the proper mathematical model* [of yield response to N]*… except that it must be a function in which yield increase or response declines with succeeding increments of fertilizer…. Since all equations are empirical, it is easier for the agronomist and economist to choose a model than it is to find an average response curve that is reliable for a range of climatic and antecedent soil and cropping conditions.* (p. 509)

With this in mind, Viets (1965) took issue with Truog's view of how total *N* uptake relates to corn crop yield. He began by asking an interesting question,

> *Does corn yielding 180 bushels an acre absorb twice as much N as 90-bushel corn?* (p. 513).

That is, is there a constant marginal product from *N* uptake? Are there constant returns to scale: are the amounts of *N*, *P*, and *K* inside a 200-bushel corn crop exactly twice the amount of *N*, *P*, and *K* inside a 100-bushel corn crop? If this is the case, then, predicting the fertilizer *N* requirement of a crop would be simple and easy following Truog's prescription. Viets (1965) looked at two relationships: (1) total *N* uptake in relation to total dry weight of stalks and leaves and (2) total *N* uptake and the yield of marketable crop (e.g. bushels of grain).

Viets (1965, p. 513) stated,

> [w]*hen the N supply is varied over a wide range from a severely limiting to a luxurious supply, a variable concentration of N relative to dry weight is usually found.*

Viets was stating that the relationship of dry weight to N supply is not linear, i.e. "kinks do not line up." A wide variation of yields was observed that among other causes could be attributed to weather uncertainties, and differences in residual N in the soil resulting from differences in N application. Viets (1965, p. 517) then concluded that,

> *… the N required in the tops per bushel at the 40—cwt yield level of grain (70 bushel) cannot be linearly extrapolated to the 80-cwt (140 bushel) level N. N intake will have to be perhaps 3 or 4 times as much instead of twice.*

Put simply, the answer to his question is no – the corn production function is not characterized by constant marginal product of *N* fertilizer. Viets's point was that because of variation in yields due to weather, soil type, and other variables, it would be difficult to come up with an accurate fertilizer recommendations algorithm using Truog's methods.

<u>2.1.2.3.1.3. The use of the minimum concentration approach</u>

Given the problem in accurately predicting yield and N content of a crop, Viets (1965) suggested that,

> [a] *better basis for arriving at the theoretical N uptake of crops under optimum conditions of moisture and supply of other nutrients might*

> *consider total solar radiation, photosynthetic efficiency of the leaves, and the experimental determination of the* ***minimum*** *concentration of total plant N that would permit the theoretical accumulation of dry weight…. The alternative is to make use of experimentally determined values of N content, both concentration and total, in relation* ***to total dry matter or yield of marketable product*** *obtained in field experiments.* (pp. 512-513, bold typeface added)

Viets (1965) supposed that other factors of production, such as weather play crucial role in crop development and agronomic field experiments. With optimal water, sunlight, and weather (i.e. if nothing is limiting or excessive), the plant takes up the optimal *N* from the soil, given that a sufficient amount is available. The amount of *N* taken up by the crop depends on the yield response function. A farmer would like to have information on how yield responds to different amount of fertilizer *N*, water and sunlight, as well as to different soil properties and weather conditions and to the interactions of these factors. Unlike Truog (1960), Viets (1965) considered that agronomic response experiments can be used to estimate crop yield response functions, and crop yield response can be best described and determined through empirical and statistical methods and models that are appropriate in the design and analysis of agronomic response experiments.

Viets (1965) was promoting here the idea of the law of the minimum as he wished to determine the minimum *N* content necessary to achieve maximum yield potential which he calls the "critical *N* concentration." Viets (1965) was pushing a critical *N* concentration approach to rationalize the analysis of field data. Gastal and Lemaire (2002, p.790) explained this approach in more detail:

> *The introduction of this concept* [minimum *N* concentration] *has allowed rationalization of the analysis of field data…. Critical N concentration is not a goal in itself to follow during crop growth, but rather is a fundamental reference at any growth stage and in any environment, which allows the determination of whether crop N nutrition is supra-optimal (i.e. actual N content is in excess compared to the N content required for maximum growth rate), or sub-optimal with respect to crop growth rate. The discrepancy between the actual N% and the corresponding critical N% at the same shoot biomass indicates the intensity of the N deficiency (or excess) experienced by a crop.*

Viets (1965) was making an assumption of a linear-plateau production function and his methods' purpose was to find the kink in the production function. That is, he was trying to find the minimum N rate that gives the maximum yield. If this is the functional form, then, as discussed earlier, input and output prices can be ignored when searching for the optimal N fertilizer rate.

The law of the minimum is also implied in Viets's (1965, p. 514) statement:

> *With high yields and the higher N contents associated with them, other factors such as supply of other nutrients begin to make themselves felt.*

Viets was emphasizing the need to do an experiment with *N* being chosen at different levels by the experimenters. He argued earlier that data was inadequate to statistically determine the critical *N* concentration of field-grown crops and hence accurately estimate yield response.

2.1.3. How is Economically optimal N rate is calculated with the mass balance approach: Stanford's hypothesis

Stanford supported Viets's (1965) results that *N* requirements vary among corn plants. However, he did not agree with Viets (1965) that the total *N* requirement of a crop cannot be accurately predicted because the total yield cannot be accurately predicted and *N* supply itself is often a factor in determining the total yield, and therefore that it is difficult to find the optimal fertilizer rate requirement of a crop. Rather, Stanford (1966) created and promoted the "1.2 Rule," which over the next several years was adopted around the world as a method of estimating economically optimal fertilizer application rates.

Stanford (1966, 1973) hypothesized that the corn plant possesses a "quantitatively definable requirement for N" which can be found by determining the "internal N requirement" associated with attainable yields. Stanford believed this internal N requirement could be estimated using the dry matter yield[6] (grain plus stover, Y^{DM}) and *N* uptake of corn (N^{up}) found in dry matter:

$$\text{Internal } N \text{ requirement} = \frac{N^{up}}{Y^{DM}} \quad (2\text{-}3),$$

which is independent of the wide range of growing conditions. Therefore, optimal *N* fertilizer rate (N^*) could be found by

$$N_F = \frac{k}{E_F}\overline{Y}^{DM} - N_s \quad (2\text{-}4),$$

where k is the internal *N* requirement of the crop, E_F is the fraction of N_F recovered by the crop, N_s is the amount of *N* obtained by the crop from the soil itself and $\overline{Y}^{DM}$ is the total dry matter of corn. Stanford's approach was to estimate the parameter k.

[6] Dry matter was defined as the dry weight of the corn's grain plus stover. It was assumed in Stanford (1973) paper that a corn has moisture content of 12 percent. Stanford (1966, 1973) assumed throughout that a corn plant's dry matter yield is always proportional to its grain yield. Therefore, it would not really matter whether to use grain yield or dry matter yield in taking the ratio of *N* uptake to yield.

In response to Viets's claim that *N* requirement is difficult to estimate, Stanford (1966) stated,

> [i]*t might appear, at first thought, that the N requirement should be defined differently for grain than for silage production. The amount of N needed to produce a given yield of grain is of primary interest to the grower. With silage, interest centers on N requirements in relation to total dry matter production.*

Although, he did not cite it, it seems possible that Stanford's approach was based on the finding of Hanway (1962) that the average dry matter of grain was 50 percent of the total dry matter. Typically, corn has a harvest index[7] of 50 percent. According to Hanway (1962, p. 145),

> [t]*he yield of total dry matter and of grain in plants from different fertility levels was proportional to the weight of leaves even though the chemical composition of the leaves was extremely variable.*

Hanway's (1962) statement suggested that during that time, grain and straw production had equal *N* amount. Hence to determine the $\bar{Y}^{DM}$, multiply the average dry matter grain by $\alpha = 2$. The α means when use *k* weight of dry grain alone, estimate of N_F is off by a factor of α. For example, if there are 1,000 pounds/acre of grain (which is 12 percent water), then its equivalent dry matter is equal to 880 pounds/acre grain and there is about 880 pounds of stover. The total dry matter is then equal to 1,760 pounds/acre. Since a bushel of shell corn contains 49.3 pounds dry matter, the total above dry matter on a per bushel basis is 98.8 pounds, hence $\bar{Y}^{DM}$= 35.7 bushels per

[7] Grain/stover ratio for maize.

acre. To get the total N fertilizer requirement of corn, the total above ground dry matter is multiplied by a factor *k* with adjustments for fertilizer efficiency and existing nutrients in the soil.

2.1.4. Stanford's empirical methodology

Stanford (1966, 1973) desired to develop a methodology for various crops, including corn, that would provide a basis for predicting the additional quantity of N required from fertilizer. Stanford stated (1966, p. 238),

> [f]*or the purposes of the present discussion, N requirement is defined as the minimum amount of this element in the aboveground portion of the crops associated with maximum production.*

That is, Stanford (1966) attempted to estimate the minimum N uptake associated with a cornfield's maximum level of yield. He concluded from his examination of the data that, for a corn plant that has achieved its grain yield potential, there is a consistent empirical relationship between the plant's N uptake and its dry matter yield. Specifically, he claimed that such a plant's N uptake will be very near to 1.2 times its dry matter yield, and that this result is consistent across a very wide range of growing conditions. Given the huge impact of Stanford's announced 1.2 Rule, however, it is important to understand that, as we will explain, Stanford derived estimates of maximum grain yield and associated N uptake by drawing *free-hand* curves through "averages of averages" of (N-uptake, dry matter yield) data. Stanford (1966) depicted the relationship of corn grain yield to the total N uptake and tested whether this relationship depends on growing conditions.

2.1.4.1 Stanford's Use of Olson's Nebraska Data

Stanford (1966) derived his 1.2 Rule by analyzing data from agronomic experiments reported in Olson, et al. (1964). Stanford's presentation of his methodology is difficult to interpret, but after having studied reports from the original sources of his data, we will attempt to provide a rigorous treatment of it. Olson, et al. (1964) designed and conducted randomized block agronomic experiments in fourteen different locations in Nebraska, which we index with $l \in \{1, 2, \dots, 14\}$. Each experiment was run for exactly one year, either 1957, 1958, 1959, or 1960. For experiment l there were B_l blocks (repetitions).

The design of each experiment was to assign one of four N fertilizer application rates to each plot in the experiment, and after harvest to measure and record for each plot the weight of the nitrogen (in pounds per acre) in the plot's dry matter (called the "nitrogen uptake"). At each experiment, three different levels of fertilizer N, 40, 80, and 160 were applied to corn for fall, spring, and summer side-dress and compared to a fertilizer N rate of zero. Some experiments had three repetitions, some had four, and some had five. There were ten plots in each block. A plot was characterized by the N fertilizer rate applied to it and the season (if any) in which the fertilizer was applied. In each block one plot was assigned an N application rate of zero, and so there was no season of application. Other plots were assigned N application rates of either 40, 80, or 160, and an application season of either fall, spring, or summer. Thus, a generic block contained set of ten plots, upon each of which was assigned exactly one of ten fertilization plans: P = {(0, *null*), (40, *fall*), (80, *fall*), (160, *fall*), (40, *spring*), (80, *spring*), (160, *spring*), (40 *summer*), (80, *summer*), (160, *summer*)}, with every plot in the block being assigned a different plan. After harvest, Olson recorded to data for each observation of his experiments: the grain yield, the dry matter yield and the N uptake[8]. Table 2.2 is a depiction of the format of the data set that would have resulted for a single experiment. A generic observation, then, can be denoted (N_{lbp}, Y_{lbp}), where N_{lbp} is the N fertilization rate used in repetition b when plan p was conducted in experiment l, and Y_{lbp} is the corresponding dry matter yield.

The raw data from a generic experiment described above is shown in figure 2.4, where it is assumed that the number of replications is four. Note that in the panel on the far left, an N fertilization rate of zero was replicated four times, and four resulting yields were recorded. The second panel from the left shows twelve data points, with four replications of N application rates of 40, 80, and 160 with spring fertilization. The other two panels could be explained similarly. With forty data points from an experiment, such as those depicted in figure 2.4. Stanford summarized the data by taking the mean across repetitions. That is, for each experiment he calculated ten points. The first point showed the experiment's mean of its (nitrogen uptake, dry matter yield) couplets resulting from no N fertilizer application, where the mean is taken over the repetitions:

$$\left(\mu_l^{Nup}(0),\mu_l^{Ydm}(0)\right)=\left(\frac{1}{B_l}\sum_{b=1}^{B_l}N_l^{up}(0),\frac{1}{B_l}\sum_{b=1}^{B_l}Y_l^{dm}(0)\right) \tag{2-5}$$

The other nine points showed the experiment's mean of its (nitrogen uptake, dry matter yield) couplets, for each season and positive nitrogen fertilizer rate:

$$\begin{aligned}\left(\mu_{sl}^{Nup}\left(N^{rate}\right),\mu_{sl}^{Ydm}\left(N^{rate}\right)\right)&=\left(\frac{1}{B_l}\sum_{b=1}^{B_l}N_{s,l}^{up}\left(N^{rate}\right),\frac{1}{B_l}\sum_{b=1}^{B_l}Y_{s,l}^{dm}\left(N^{rate}\right)\right),\\ N^{rate}&=40,80,160;\ \ s=fall,spring,summer.\end{aligned} \tag{2-6}$$

[8] Nitrogen uptake or the total nitrogen content of the mature grain and corn stover was determined by Kjeldahl distillation (Olson, et al. 1964).

After examining the ten summary points for each of the fourteen experiments, Stanford divided the set of fourteen experiments into three subsets of experiments, the first which included four of the experiments, the second which included six experiments, and the last, which included the remaining four experiments to show the relationship of grain yield and total *N* uptake. He then summarized each group's summary points by taken their mean across experiments in the group. Letting G_g be the set of experiments in group g = 1, 2, 3, and letting N_1 = 4, N_2 = 6, N_3 =4, he found ten (nitrogen uptake, dry matter yield) points for each group. A group's first summary point was the average of the group's experiments' no-fertilizer points:

$$\sum_{l \in G_g} \frac{1}{N_g} \left(\mu_l^{Nup}(0), \mu_l^{Ydm}(0) \right), g = 1,2,3. \tag{2-7}$$

The group's other nine summary points were the averages of the group's experiments' (nitrogen uptake, dry matter yield) summary points, one for each season and nitrogen fertilizer rate:

$$\sum_{l \in G_g} \frac{1}{N_g} \left(\mu_{sl}^{Nup}\left(N^{rate}\right), \mu_{sl}^{Ydm}\left(N^{rate}\right) \right),$$
$$g = 1,2,3; \qquad N^{rate} = 40,80,160; \qquad s = fall, spring, summer \tag{2-8}$$

The resulting thirty "summary of summary points" are shown in figure 2.5 (left panel), which comes from Stanford (1966).

The figure also gives an idea of how Stanford chose which experiments to place in which groups: quite subjectively. Except for an obvious outlier, the thirty points seem to have a linear relationship. But Stanford chose his groupings so as to have each group's ten summary-of-summary points more or less reach an *N*-uptake level at which the dry matter yields plateau. Stanford reported no statistical methods used to make the groupings and test whether one group is significantly different from the other group. Not did he conduct statistical estimations to parameterize the yield curves that he drew. From the figure, it might be argued that experiments in groups 1 and 3 were made in order to generate “high” (nitrogen uptake, dry matter yield) points in group 1 and “low” (nitrogen uptake, dry matter yield) points in group 3. But the summary of summary points in groups 1 and 2 seem to be chosen on a subjective, if not overtly convenient basis. Moreover, in his 1973 paper, Stanford did not include group 2 in his analysis (Figure 2.5, right panel). Why the “medium yielding” group was excluded from the 1973 paper is not clear, and there is no way to know at this point how consistent the results of his procedure run on the medium-yielding points would have been with the rest of Stanford’s story. Stanford took the average-of-averages of the data that he had calculated in his 1966 paper, and ran OLS regressions through the point, assuming a quadratic functional form. He reported the estimated coefficients of his regressions, but gave no indication of their statistical significance. In any case, given his method of taking averages of averages, how the coefficients should be interpreted is unclear.

2.1.4.2 Stanford’s Use of Southeastern U.S. data

To test whether the N requirement for maximum yield was affected by growing conditions, level of yield attained, corn variety, and other variables, Stanford (1966) used the experimental results of Pearson et al (1961), in addition to the data provided by Olson (1964). Pearson et al (1961) reported results from field experiments in 1955 at three locations in Alabama, one in Georgia, and two locations in Mississippi, and in 1957 at one location in Georgia. Stanford (1966) used the data from the Mississippi and Georgia experiments, but for unexplained reasons ignored data from the three Alabama experiments. The detailed summary of data Stanford used is presented in Table 2.3.

I tried to replicate figure 2.6 in Stanford (1966) paper (Figure 2.4) based on the published data in Pearson et al., (1961)[9]. Figure 2.7 was based only on published spring data[10]. The curves were fitted using a quadratic function model. The results from the Alabama data suggest that the maximum yield was achieved at more than 1.6 times *N* concentration. Also, the dry matter yield curve from the Pratville, Alabama data (represented by the sky blue curve) is low compared to the yield curves obtained from the data in field experiments in Mississippi and Georgia.

Stanford concluded that the *N* requirement of a crop is the product of the maximum attainable yield of dry matter (grain plus stover) and the critical internal nitrogen concentration, 1.2 percent. He claimed that the critical concentration for corn (on nitrogen) was unaffected by variety, location, climate, or level of attainable yield, and remained essentially constant at 1.2% based on the results in Fig 3. Stanford's ideas continue to influence the literature as Gastal and Lemaire (2002, p. 790) opine,

> [c]*ritical N concentration is not a goal in itself to follow during crop growth, but rather is a fundamental reference at any growth stage and in any environment, which allows the determination of whether crop N nutrition is supra-optimal or sub-optimal with respect to crop growth rate*.

2.2. The Importance of the Form of the Response Function

University research personnel and extension agents have long made recommendations in attempts to influence farmers' fertilizer decisions. Despite the prevalence of the 1.2 Rule in

[9] We assumed moisture content of 12 percent, harvest index of 50 percent and a bushel of corn is equal to 56 lbs. of corn.

[10] Note that in experiments in Alabama and Mississippi, fall applications supplied *N* at 75 or 100 pounds per acre only, and the yield curve was defined by spring applications of 0, 50, 100, 150, and 200 pounds per acre. This means a yield curve for fall applications is drawn based on the spring application at different rates.

nitrogen fertilizer application recommendations, there was a large literature which took an alternative approach. “Economically optimal *N* fertilizer recommendations” are obtained by fitting yield response functions to crop yield an input application rate data from controlled agronomic experiments (Babcock, 1992; Lanzer and Paris, 1981; Mooney et al., 2008), and hence can vary depending on the functional form used to estimate yield response functions. The use of the most appropriate functional form to estimate crop response models is important in agronomic and economic research. Both agronomists and agricultural economists have made considerable efforts pursuing various methods of estimating optimal *N* fertilizer rates in agricultural production. Note that these two literatures were developed side-by-side, neither influencing the other much.

2.2.1. Crop Scientists’ View of Crop Response

Crop scientists use fertilizer experiments designed to provide response data amenable to economic analyses. Unfortunately, most crop scientists have had little access to microeconomic theory and marginal analysis. I will show that they circumvented this problem by in effect using very restrictive assumptions about the functional form of the crop response. Most commonly, these assumptions were not stated explicitly in their reported research. By assuming von Liebig response functions, fertilizer recommendations were made simple; the economically optimal application rate is the minimum rate at which yield reaches its plateau. That is, the optimal rate does not depend on input and output prices[11] or marginal analysis. I am not claiming that farmers do not respond to input and output prices nor the production function is von Liebig. I am simply giving details about the economic implications of early agronomists’ assumptions about the production function. The von Liebig production function is only used

[11] Except for the extreme case of a corner solution, in which prices lead to zero production.

here to explain why early agronomists did not think they needed to worry about prices when estimating optimal N rates.

2.2.2. *Economists' View of Crop Response*

Since the 1950s a great effort has been reported in the agricultural economics literature of trying to obtain accurately estimated crop response functions. One principal goal of such research is to help farmers make better decisions on fertilizer input and output levels in crop production. Production functions are used in economic analyses of crop response to fertilizer. Economic decision rules are used to determine the profit maximizing level of fertilization. The profit maximization problem is given as

$$\underset{y}{Max}\ p \cdot f(x) - w \cdot x \qquad (2\text{-}9),$$

where p is the output price, $f(x)$ is the crop production or response function, w is the input price and x is the input variable. The well-known first order condition to this maximization problem is that nitrogen fertilizer should be applied at the rate at which its marginal product equals the price ratio: $f'(N^*) = w/p$. Ignoring complications like uncertainty and risk, the economic method of estimating N^* is to estimate the crop response function, assume levels of w and p, and find where first-order conditions are satisfied.

Economists recognize that as long as the response function is continuously differentiable, then given a non-zero price ratio w/p, there is a difference between the yield maximizing and profit maximizing input levels. Maximum yields are seldom associated with maximum profits. To estimate economically optimal N levels, input and output price data are applied to estimated crop response functions to estimate economically optimal N levels. Economists often consider the response between yield and fertilizer to be smooth and assume diminishing marginal product (conforming to the Law of Diminishing Returns).

The primary methodological challenge in estimating crop response functions is to make a proper choice of the algebraic form of the yield response function. For several decades the work by Heady and Dillon (1961) promoted the use of polynomial forms for agriculture production models and led the way in developing applying the economic approach to fertilizer recommendation. The use of polynomial forms (such as quadratic) in the approximation of the crop response function is appealing because (1) it is easy to manipulate (Grimm, Paris, and Williams, 1987) and (2) it is accompanied by computational simplicity and high fit (Swanson, et al., 1973; Ackello-Ogutu, Paris, and Williams, 1985 (hereinafter referred to as APW, 1985)).

2.2.3. The question becomes: What is the correct functional form?

Many researchers have assumed that response functions can be best estimated using polynomial functional forms. Other studies have considered the limiting nutrient response functional form. Despite significant research efforts, no consensus has been reached about which functional form best represents corn response to *N* fertilizer, chiefly because insufficient data has been generated.

2.2.3.1. When agricultural economists started to examine the problem, thiey assumed "smooth and concave" functional forms

2.2.3.1.1. Started with Heady and Pesek's straightforward approach, developed at the same time with Truog, Viets

Heady and Pesek (1954) were early and significant contributors to corn-fertilizer response function research, which aimed at discovering more about yield response to increasing nitrogen application rates and why it differs between sites. Heady (1957, p. 249) claimed that,

> [p]*hysical scientists have concentrated on providing point estimates* [i.e. production coefficients] *which are most practical; and they have been efficient in*

doing so. At the same time, however, they have been carrying on experiments relating to the phenomena concerned but have employed models which suppose the observations to be discrete. Their general approach, in which a large proportion of Land Grant College research resources have been invested, generally provides a ***notion of a very few points*** *on the production surfaces; and ordinarily do not lend themselves to economic interpretation by farmers who must use them.*

2.2.3.1.2. Like crop scientists, agricultural economists too did agronomic experiments but they designed them for economic analysis and assumed smooth functional forms in their econometric analysis

Heady and Pesek (1954) argued that to increase knowledge of the yield-response-to-fertilizer function, fertilizer recommendations should be based on experiments with at least two variable nutrients[12] and several functional forms should be applied to the data in the process of response function estimation. To apply this approach, they conducted agronomic experiments that included a wide range of fertilization rates in small-size plots, and replicated them to control the magnitude of standard error. They fit five models to the yield and input level data to estimate the rates at which the marginal increase in grain value would equal marginal N fertilizer cost, which is generally the nitrogen fertilizer input price, which we denote *w* (Heady et al., 1955). Thus, they estimated the economic optimum nitrogen rate (EONR) as the which would maximize the return to N per area-unit. Heady and Pesek (1955) advocated this method and suggested it should be adopted for general use. In the 1960s, many studies followed which applied polynomial functional forms to data from experiment-station and on-farm agronomic

[12] Agronomists' usually estimate a single-variable nutrient production function.

experiments (Desai and Doshi, 1962; Walker, et al., 1963; Walker and Carmer, 1967; Anderson, 1968; Rouse, 1968; Khare et al., 1968; Fuller, 1969).

According to Hutton and Thorne (1955), Heady and Pesek's agronomic-economic collaboration was

> *unusual in that the field experimentation was planned to fulfill the requirements of analysis amenable to economic interpretation.* (p. 117)

At the same time, they criticized Heady and Pesek's proposed method as simply being a "methodological exercise," not of much general use. They claimed that Heady and Pesek's results "are of trivial economic importance." The economic loss from not using the economic optimum of fertilizer combination predicted from their regressions would only represent a very small percentage of the estimated gross income less fertilizer cost (e.g. from 0.2 percent – 0.6 percent) and that the large number of small plots used in their experiments was "wasteful" agronomic research since they would have come up with the same conclusion with fewer experiments (i.e. 114 vs 34 field plots). Hutton and Thorne (1955) emphasized the need for further empirical investigation, and that the methods used must account for interaction effects between the nutrients at rates of application that are relevant to the interest served by the research. Despite of the criticisms, the methodology of Heady and Pesek continued to be used throughout the 1950s (Doll, 1972), and related research has continued through this day.

2.2.3.2. Literature assuming smooth functional forms and literature assuming von Liebig-type functional forms

Studies that assume smooth functional forms include Neeteson and Wadman (1985), Bullock and Bullock (1994), Dawe and Dobermann (1996), Raij and Cantarella (1997), Ruffo, et al. (2006), Finger and Hediger (2008), Mooney and Roberts (2008), and Kachanoski (2009). There are also numerous studies that assume von Liebig's model (e.g. Babcock and Blackmer, 1994; Kreuz, et al., 1995; Chambers and Lichtenberg, 1996; Babcock and Pautsch, 1998; Lark, 2001; Makowski and Wallach, 2001; Kaitibie, et al., 2007; Zhang, et al., 2007; Tembo, et al., 2008; Marenya and Barrett, 2009). Again, a fuller literature review will be discussed in the next chapter of this dissertation.

2.2.3.3. Active debate over functional forms of yield response.

An active debate surrounding crop response models has been centered around which functional form provides a better representation of crop response to different *N* fertilizer level. Since Cate and Nelson's (1971) proposal formally reintroduced into the crop response analysis the von Liebig's law of the minimum and the notion of maximum plateau, numerous studies have been performed to compare the von Liebig with polynomial models. Perrin (1976), Lanzer and Paris (1981), and Grimm, Paris, and Williams (1987) concluded that linear plateau models performed as well or better than polynomial models. APW (1985) emphasized that polynomial functional forms are often to blame for recommending inefficient use of fertilizer inputs.

In 1990, Frank, Beattie, and Embleton tested the Mitscherlich-Baule (MB) form against the von Liebig and quadratic. The MB allows for both factor substitution and plateau growth. The authors recommended the use of the MB form based on pairwise J-tests and P-tests. This was then challenged by Paris (1992) who estimated a non-linear von Liebig model against an MB, quadratic, square-root, and linear von Liebig using a switching regression model, based on the technique outlined in Maddala and Nelson (1974). He concluded that a plateau function is a more appropriate fit than polynomial specifications. Chambers and Lichtenberg (1996), however, found evidence of yield plateaus, but also found input substitutability, and hence concluded that the von Liebig approach was only appropriate under certain circumstances and for particular crops. Following Chambers and Lichtenberg (1996), Berch, Geoghegan, and Stochs (2000) presented a non-parametric estimation of right-angle isoquant production functions. They disagreed with Paris (1992) and found that the von Liebig production function to provide a poor fit and little evidence for right-angled isoquants.

In 2002, Holloway and Paris revisited the von Liebig by reexamining five samples of experimental data and by combining frontier methods with the von Liebig methodology using Bayesian techniques. They acknowledged recent nonparametric tests rejecting the von Liebig model, but were unable to reconcile results from parametric and nonparametric methods. Tembo et al. (2008) utilized the switching regression model used by Paris (1992) but added an uncorrelated random effect for year and a stochastic plateau. They found this model provided a better fit to data from a long-term experiment than did the switching regression model of Maddala and Nelson (1974).

2.3. Critiques of the 1.2 Rule

In this section, I discuss the economic and statistical issues arising in the design and analysis of Stanford's experiments, and the credentials they provide to Stanford's 1.2 Rule as the basis of offering recommendations to corn farmers about fertilizer management.

2.3.1. The yield potential method makes economic sense only under very restrictive assumptions about the form of the yield response function

2.3.1.1. von Liebig/Leontief functional form

A key weakness or inconsistency in Stanford's approach as it has been applied is that sometimes the "yield goal" approach has been used, while other times "yield potential" approach has been used. In Stanford's approach, at least strictly speaking, the use of "yield goal" to maximize profits makes little economic sense. After all, if a farmer's goal is to maximize profits, he cannot determine how to maximize profits by first examining which yield will maximize profits. Conceptually more tenable may be the claim that if a farmer has insights into the maximum yield he can achieve (the "yield potential), and this knowledge might somehow offer information about the optimal N rate. To have an idea of the "yield potential," a farmer would need to have some idea about the maximum yield attainable on his field, as suggested by Viets (1965). For the yield potential approach to make economic sense, the production must satisfy two restrictions: (1) the production function is von Liebig, i.e. there is a kink in the function, so that input and output prices do not affect the (interior) solution to the profit maximization problem[13]; and (2) the kinks of the von Liebig response curves for different weather, soil type, and other factors of production "line up" on a ray out of the origin with slope 1.2 (Figure 2.8). Under these two conditions, the relative ratio of input and output prices will not matter and the farmer can maximize profits by finding the lowest level of N fertilizer at which the response function reaches its plateau height. Based on figure 2.1b, if indeed the production function is von Liebig, the farmer will either choose 0 or $\overline{N}$ amount of input to maximize profit. Note that I am not claiming here that the von Liebig is the correct production function. The von Liebig function is only used as a starting point to test the validity of the 1.2 Rule.

In his empirical analyses, however, Stanford (1966, 1973) offered no formal statistical evidence about whether the experimental data provided evidence that these two restrictions were satisfied. Stanford did not even draw von Liebig curves through a scatter plot of the data. It was not obvious at all that the data he used shows anything like a "kink" and if there are indeed

[13] Unless, as stated before, the maximization problem has a corner solution.

"kinks", that they are located where Stanford's free-hand curves show them to be. It could not be possible for him to figure out the location of the kink because only three *N* rates were used in Olson's experiments that provided him his data. As shown in figure 2.6 previously, Stanford's claimed calculated N concentration ratio ranged from 1.1% to 1.3%. There was no perfect linear relationship in the data between his claimed N levels at the beginnings of claimed yield plateaus. By looking at all the data in the figure 2.6, it would be difficult to be able to say much about the functional form of the plateau and how variable such responses are over years, fields, and cropping seasons.

2.3.1.2. And, "kinks must line up"

Even if agronomic theory makes von Liebig technology a plausible representation of true response functions (and it is not clear that it does), it remains unclear why the kinks should all lie on a common ray from the origin in an *(N,y)* diagram. If kinks do not line up, the critical *N* concentration of plant's dry matter will vary (Figure 2.9). In this case Stanford's 1.2 rule basis of fertilizer recommendations misleads. Thus, it is important to test statistically if the kinks of the von Liebig response curve line up on a common ray.

2.3.2. When prices, risk, and uncertainty matter

Prices and the form of the production function together fundamentally affect optimal input application rates. Even though, strictly speaking, Standord's 1.2 Rule, which does not consider prices, makes economic sense only if the true response function takes on a von Liebig function form. Stanford (1966) created his 1.2 Rule by drawing smooth curves free-hand through a scatter plot of "averages of averages" data (Figures 2.2 and 2.3, pp. 244-245). The curves he drew appear much like quadratic-plus-plateau curves, not von Liebig-type curves with kinks. In his 1973 article, he took the same averages-of-averages data, assumed a quadratic functional form, and ran a regression model to estimate the coefficients.

In the case of a quadratic-plus-plateau or quadratic functional form, profit maximization requires information on (1) output price; (2) fertilizer input price; and (3) the marginal product of each increment of fertilizer. An optimum fertilization level (N^*) is attained when the marginal product of fertilizer is equal to the fertilizer price and output price ratio (Figure 2.10a). This is where the slope of the production function (i.e. marginal product) is tangent to the (w/p) line. Because of the concave shape of the quadratic response function, any change in the input rate away from N^* would lead to a loss in profits. If a farmer applies fertilizer at a rate lower than N^*, he can still increase profits by adding more N fertilizer. If a farmer applies more than N^*, the cost of adding another input is greater than the return derived from its use. Thus, as illustrated in figure 2.10b, the optimum rate of fertilization changes with the price ratio. Assume that $p > 0$, $w > 0$, $p'' > p^* > p'$, and $w'' > w^* > w'$. If the price of corn increases from p^* to p'' (or w^* decreases to w'), the farmer is encouraged to use more N fertilizer to grow more corn. As long as the revenue the farmer receives from the extra output from increasing N is greater than the cost of that unit of N, fertilizer use should be increased. That is, the farmer will stop production at the point at which the value of the marginal product of N equals w. With the new (w/p), the new economically optimal N rate will be N'. On the other hand, if the price of corn decreases from p^* to p'' (or w^* increases to w'), more units of corn must be exchanged for a unit of fertilizer, the profit-maximizing farmer will use less N. The new economically optimal rate is now N'. In 2008/2009 when fertilizer prices spiked given high crop prices, the fertilizer consumption (kg/ha) of farmers decline (Figure 2.10c).

So far I only assumed that the farmer is risk-neutral and the response function is non-stochastic. Given mean growing conditions, a risk-neutral farmer applies fertilizer at a higher rate as long as the expected gain in profit from the increased yield in a good state of nature is higher than the expected loss in profit from wasted fertilizer in the bad state of nature. Neither of these assumptions are necessarily realistic and these may lead to inappropriate *N* fertilizer rate recommendations. Farmer's input decisions, including fertilizer use, are typically influenced by risks (e.g. risks from pests and other unmanageable inputs) and stochastic factors (e.g. soil variability, weather). That is the recognition that the nutrient choice does not determine mean response alone. And given farmer objectives other moments of the distribution might be important. How risk-aversion affects nutrient management depends on whether fertilizer is seen as a risk-reducing or risk-enhancing input. Given weather uncertainty, if fertilizer is seen as risk enhancing, a risk-averse farmer applies fertilizer at a low rate than risk-neutral farmers (Just and Pope, 1979). In cases when fertilizer can be risk reducing, risk aversion should generally result in higher application rates. Some important works in the literature are Day (1965), Just and Pope (1979), Antle (1983), Nelson and Preckel (1989), Henessy (2011), and Du, Hennessy, and Yu (2012).

2.3.3. Other statistical and econometrics issues

To better understand the yield response function, the farmer not only requires information about how yield will respond to different rates of application of N fertilizer. Other factors of production such as managed inputs (e.g. seed, labor) as well as stochastic factors such as soil variability, weather, insects, diseases, residual fertilizer and nutrients in the soil, and the interactions between the managed inputs and stochastic factors must also be taken into account. Stanford, however, simplified his analyses by assuming that factors of production such as weather variability and differences in soil types are nonrandom and, by ignoring completely the year and site-specific effects on the experimental data he used. Because of this econometric issue, his claim that the critical concentration for corn (on nitrogen) was constant at 1.2% given a wide varying growing condition might not be valid. It is important to emphasize that Stanford (1966) only utilized field data based on few years (1957-1960) and few locations (experimental sites in Nebraska, Mississippi, and Georgia) to account for variability in managed and non-managed factors that serve as arguments in the yield function. Soil nutrient composition in a field tends to vary stochastically from site to site and year to year and hence heterogeneity may exist. Even management practices, which the researcher for the most part can control, are subject to measurement error, human error, and several other sources of variation (Tembo, et al., 2008).

2.3.4. Inconsistencies in later interpretations of Stanford's findings

Two of the most important components of Stanford "1.2 Rule" are the yield goal and N requirement of crops (i.e., the *N* at the "kink" in the response function). In the past decades, studies have used various interpretations of the concepts of the "yield goal" and "*N* requirement." In the latest studies, yield goal has ended up being interpreted as the maximum possible grain yield. While the "*N* requirement" was defined as the minimum amount of *N* fertilizer needed to achieve that yield. But these interpretations are not consistent with the interpretations that Stanford was making when he established in his 1.2 Rule. Stanford defined "yield goal" as total amount (in bushels per acre) of *grain and stalks* of corn that a farmer wishes to grow, while "*N* requirement" is the dictated by the *N* uptake (in pounds per acre) in the plot's dry matter.

2.3.5. Critiques of 1.2 Rule in the literature

A significant number of research studies suggest that the yield goal approach results in over-fertilization of corn (e.g., Vanotti and Bundy, 1994). Lory and Scharf (2003) investigated 298 previously reported experiments in five Corn Belt states in the U.S., and estimated that the recommended N fertilizer rates determined by the yield goal approach to implementing Stanford's 1.2 Rule exceeded the EONR by an average of 80 lb/acre.

Other studies also cast doubt over the appropriateness of the yield goal-based approaches in N fertilizer recommendation. They suggest (1) poor relationships between 1.2 Rule-based recommendations and the EONR observed in N rate response trials (Blackmer et al., 1991; Vanotti and Bundy 1994a; Vanotti and Bundy 1994b; Fox and Piekielek, 1995; Kachanoski, et al., 1996; Lory and Scharf, 2003); (2) uncertainty about how yield goals should be determined; and (3) use of inadequate or inappropriate adjustments for nonfertilizer N sources in yield goal approaches (Sawyer et al., 2006).

2.4. Understanding Stanford's 1.2 Rule in Its Historical Context

While the critiques of the development and use of Stanford's 1.2 Rule discussed above are valid, the assumptions and motivations that led to them must be understood in their historical contexts. Although both agronomists and economists during Stanford's time already recognized that the problem of optimal *N* rate estimation depends both on agronomic field trials and on economic analysis, it is understandable why he still did not do enough systematic effort to incorporate sound microeconomic theory to his analysis. This is because during his time, agronomists' and soil scientists' primary concerns were only to motivate the farmers to use fertilizer in roughly "reasonable" quantities. Data available at time provided very little information about any particular field's "true" response function, and so combining that information with rigorous microeconomic analysis including prices, risk considerations, etc., was very unlikely to produce useful results. Agronomists may also have been hesitant to apply economic considerations as part of their research because they lack understanding of the mathematical procedures and have been dismayed by the technical jargon developed and used by economists (Baum and Heady, 1957). This can make communication difficult and may result to confusion. Therefore crops scientists had few incentives to learn and apply economic theory in their research. Rather, their chief interest lay in more basic physical science to better understand how plant growth depends on various managed and unmanaged factors (Ko, 1960). Furthermore, during those years fertilizer was relatively inexpensive (USDA-ERS, 2012), so losses from over-fertilization were not likely to be large. For these reasons, early on fertilizer experiments were designed and conducted primarily by agronomists, with economists taking little interest (Ko, 1960). Agronomists and economists over-specialized on their respective fields and such practice did not encourage much interdisciplinary research work.

Most importantly, the mathematical and statistical education of agronomists and soil scientists during that time limited Stanford's methods and analysis. The advances in economic and econometric practices and computing technology needed to express the law of the minimum and to test what functional form best represents the crop response to *N* were not yet widely available given only a small amount of experimental data set. Today, agricultural economists have been sufficiently trained in advance statistical and econometric methods and have access to advanced computer software packages that are necessary for sound economic analysis.

Despite of all its limitations, perhaps the most valuable contribution of Stanford to fertilizer recommendation was that with the 1.2 Rule, farmers could determine on their own, if only *very* roughly, their crops' fertilizer rate requirements, using only the yield potential of their fields and of the soil properties through soil testing labs to know the amount of N_S. The 1.2 Rule can accommodate local soil *N*-cycle processes and local N sources, and has the potential to account for year-to-year variations in soil *N* processes making the *N* fertilizer rate recommendation field-specific.

2.5. But Still, How Could So Little Lead to So Much?

In past decades, most land grant universities and soil testing laboratories provided *N* fertilizer recommendations based on Stanford's 1.2 Rule, and many continue to do so. It was believed and promoted that this rule would give a farmer maximum profits. For example, Laboski and Bundy (2005) claimed that

> [t]*he yield goal method appeared to work in the 1970s when yield levels were lower than today.* (p.1)

These fertilizer recommendations using the yield goal approach are also published and widely used as the technical criteria for nutrient management regulatory policy, which often view university recommendations as a vehicle for achieving environmental objectives (Bundy, 2006). Given this, nutrient management is not only important in improving crop yields and achieving maximum profits, but more so in sustainably using natural resources.

Only recently have most land-grant universities admitted that the use of Stanford's 1.2 Rule is faulty. Camberato (2011) even remarked that

> [r]*ecent research has shown the yield-goal based N recommendations of the last 40 years are not useful for making N recommendations. In other words the amount of N needed to maximize yield is not related to yield.* (p. 6)

The 1.2 Rule did not receive any serious analytical scrutiny and empirical testing and almost no one did any follow-up research to verify Stanford's results. Only recently have university extension system begun to move away from the 1.2 Rule's algorithm.

Advocates of 1.2 Rule recognized the economic importance of fertilizer management. For example, Nafziger et al. (2004, p.1) mentioned,

> [m]*aking N rate recommendations for corn has been one of the most economically important goals of publicly funded crop production and soil fertility personnel and programs over the past five decades*[.]

But the limitations discussed and scrutinized in this paper suggest that sound economic theory, data from high-quality agronomic experiments, and proper statistical techniques were never combined in the development of economically optimal fertilizer recommendations. Although agronomists and agricultural economists both recognized the importance of interdisciplinary research among them, little was ever conducted.

To build confidence in fertilizer recommendations, every aspect fertilizer recommendation development, formulation, and delivery requires careful examination, including the running of field trials, the choice of response functional form, the type of data used for estimation of the economic optima, and the empirical and statistical analysis used. Clearly, this is an area of research in great need of interdisciplinary research among agronomists and agricultural economists to integrate all the relevant information to a farmer's decision making with economic analysis. Failure to systematically do so resulted in inappropriate fertilizer recommendations and unsustainable use of resources over the past 40 years. Therefore, it is imperative that crop scientists and agricultural economists work side-by-side to develop better means of forecasting how to adjust *N* fertilizer levels that raise farmers' profits. It is not clear that the agricultural public, whose political support for agricultural research is vital to its government funding, fully appreciates the difficulty involved in accurately estimating economically optimal N rates.

2.6. Recent changes in university recommendation algorithm

To address the previous critique of 1.2 Rule about the shortcomings of Stanford's 1.2 Rule, several states in the Corn Belt have in very recent years abandoned the yield goal approach and moved toward more data-driven recommendations that are sensitive to *N* and grain prices. In 2004, university agronomists and soil fertility specialists developed "a new philosophy" in *N* recommendations, based on a regional approach. The underlying premise of the new philosophy is to provide rate guidelines based directly from the results of many nitrogen response trials and flexibility for producers in addressing risk and price fluctuation. To avoid confusion of farmers as to what the right *N* rate to apply in their field, each state analyzes yield and *N* rate data following the new philosophy but using data from each state individually taking into account local soil and climate variability. The approach is believed to be simpler and more realistic as it provides relatively generalized recommendations based on multi-location producer *N*-response studies performed on a regional basis (e.g., Nafziger et al., 2004, Laboski and Bundy, 2005; Sawyer et al., 2006, Camberato et al., 2011).

To use this approach, a series of nitrogen rate experiments is conducted over different locations and years. Most land grant universities usually conduct small-plot, long-term experiments or trials across a range of growing conditions but data can also come from on-farm, field-scale, replicated *N* rate studies conducted by farmers. Yield data are then collected at replicated *N* rates from many *N* rate trials from each state over relevant locations and years. A yield response to *N* function (usually a quadratic or quadratic-plus-plateau) is then estimated for each site-year. The return to *N* for each site-year in the data set is determined, i.e. the total dollar return from *N* fertilizer or the EONR from marginal returns.[14] The average return to *N* at each *N* rate is then calculated using all of the data for a given crop rotation. The *N* rate with the highest average return to *N* is called the maximum return to nitrogen (MRTN), which can also be referred as the EONR. Nitrogen rates with net return within $1.00/acre of the MRTN provide a range of *N* rates recommended to producers with similar profitability.[15] The idea of giving a range of profitable *N* application rates instead of a single *N* rate to farmers is to give them flexibility to deal with economic changes (e.g. fluctuation in *N* fertilizer prices). An example of this approach is used in Illinois, Indiana, and Wisconsin.

Recent studies argue that MRTN-based recommendations are better that the yield goal approaches because for one the new approach takes into account the inherent differences in *N* provided by soils can vary substantially (i.e. less *N* is recommended on productive soils and more *N* on poorly drained soils). The new guidelines also account for diminishing marginal returns of grain yield by using quadratic-plus-plateau response function instead of straight-line

[14] Calculate the yield increase over the yield obtained when 0 lb *N*/acre is applied for every 1 lb *N*/acre applied. Multiply yield increase by price of corn and subtract the cost of *N*. Do this for all the N rates.
[15] Proponents of the new approach suggest that net returns to *N* tend to be rather flat on top.

response function[16] (Camberato, 2011). In addition, the MRTN approach considers specific responses of each site in determining the optimum *N* and net return rather than average response (Sawyer, 2006). Guidelines in fertilizer recommendation can also be grouped according to soil and climatic conditions, cropping systems (corn-corn or corn-soybean), production characteristics (high yield potential, medium yield potential, and low yield potential), and other factors affecting N-crop response.

One major limitation of this approach is that it requires the use of a large amount of nitrogen response data collected from the US Corn Belt. More so, the new approach cannot be used to predict site-specific requirements, but can only provide the *N* rate that reflects the probability of achieving expected economic return across a range of locations and period of time (Sawyer et al., 2006). Most of the data analyzed from each state come from *N* response trials in small plots of university agronomists and extensions but not from farm *N* rate trials on farmer's fields. This implicitly suggests that extension system tries to push farmers into using fertilizer recommendations that might not be suitable in their particular field since there is no consideration on farmer's specific needs, management, preferences, resources, and environment. The new approach also implicitly assumed that the difference in MRTN between states is attributable to differences in soil and climate. Hence there is no statistical test performed to know what makes the difference in N application rates recommended between and among states

2.7. Where to from here: On-farm experimentation

In light of the issues above, *N* fertilizer rate guidelines need to be more farm-specific to account for farmer's specific crop growing conditions, crop and soil management, and climate –

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[16] This allows the calculation of EONR from N fertilizer and grain prices, which results in a recommendation than can be easily adjusted with changing economic conditions

which can vary greatly among fields, seasons, and years. One way to achieve this is to do on-farm experimentation. On-farm experimentation is replicated, scientifically valid research with field trials established and managed by the farmers with field-scale equipment. Properly designed, on-farm experiments can account for the effects not just of different crop growing conditions but also the effects of management options on crop yields. On-farm experiments have been gaining considerable interest due to the availability of modern tools of information technology in such as yield monitors, geographic information systems (GIS), and remote sensing. The use of these modern technologies has greatly broadened the scope of obtaining data from farmers' fields.

Table 2.1. Fertilizer *N* recommendations

STATE	Soil Classification	Yield Goal 120 bu/ac Recommendation (lb -N/ac)	Ratio (lb-N/bu)	Yield Goal 160 bu/ac Recommendation (lb -N/ac)	Ratio (lb-N/bu)
CORN					
Tri-State (Michigan, Ohio and Indiana)	All	136	1.14	191	1.19
New York	Low N supply	145	1.21	218	1.37
	High N supply	106	0.88	180	1.13
Wisconsin	2-4% OM	120	1	160	1
Minnesota	<3% OM	132	1.1	176	1.1
	>3% OM	92	0.77	136	0.85
Illinois	All	144	1.2	192	1.2
Ontario					
South-West (Preplant)	All	160	1.33	178	1.11
South-West (Sidedress)	All	131	1.09	142	0.89
West-Cantral	All	102	0.85	120	0.75
East	All	106	0.88	156	0.98
SOYBEAN					
Tri-state	All	106	0.89	161	1
New York	Low N supply	145	1.21	218	1.37
	High N supply	106	0.88	180	1.13
Wisconsin	2-4% OM	80	0.67	120	0.75
Minnesota	<3% OM	92	0.77	136	0.85
	>3% OM	52	0.43	96	0.6
Illinois		104	0.87	152	0.95
Ontario					
South-West (Preplant)	All	147	1.23	165	1.03
South-West (Sidedress)	All	118	0.98	129	0.81
West-Cantral	All	75	0.63	93	0.58
East	All	79	0.66	129	0.81

Source: Janovitek (2011)

Table 2.2. Data set for a single experiment

Obs	Experiment	Season	*N* Application Rate	Repetition	Grain Yield	Dry matter yield	N uptake
1	1	null	0	1			
2	1	null	0	2			
3	1	null	0	3			
4	1	null	0	4			
5	1	Fall	40	1			
6	1	Fall	40	2			
7	1	Fall	40	3			
8	1	Fall	40	4			
9	1	Spring	40	1			
10	1	Spring	40	2			
11	1	Spring	40	3			
12	1	Spring	40	4			
13	1	Summer	40	1			
14	1	Summer	40	2			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
39	1	Fall	160	3			
40	1	Fall	160	4			

Table 2.3. Southeastern U.S. data used by Stanford (1966)

LOCATION	SPRING-applied N as ammonium nitrate at pounds shown per acre									
	0	30	50	60	90	100	120	150	200	240
Brooksville, Ms (Houston clay)										
Yield (Bushel - only grain)										
1956	36.1		40.5			61.8		62.9	62.6	
1957	41.9		74.3			81.2		90.3	88.0	
1959	21.8		31.4			53.1			78.3	
Average	33.3		48.7			65.4		76.6	76.3	
N uptake (grain + stover (lb/acre))										
1956	34.0		45.0			56.0		72.0	79.0	
1957	56.0		96.0			109.0		142.0	153.0	
1958	40.0		49.0			83.0		100.0	130.0	
1959	26.0		40.0			59.0			116.0	
Average	39.0		57.5			76.8		104.7	119.5	
Poplarville (rusty sandy loam)										
Yield (Bushel - only grain)										
1956	33.7		71.9			75.0		85.8	88.5	
1957	21.2		37.8			42.5		41.7	43.8	
1958	13.5		40.6			64.8		75.0	68.1	
1959	6.7		39.4			57.4			75.4	
Average	18.8		47.4			59.9		67.5	69.0	
N uptake (grain + stover (lb/acre))										
1956	31.0		69.0			78.0		99.0	106.0	
1957	25.0		43.0			63.0		68.0	72.0	
1958	19.0		55.0			83.0		102.0	98.0	
1959	9.0		42.0			61.0			111.0	
Average	21.0		52.3			71.3		89.7	96.8	
Watskinville, Ga (cecil sandy loam)										
Yield (Bushel - only grain)										
1957	39.3	71.9		90.7	101.9		99.3			102.0
1958	33.0	66.4		78.6	92.2		107.8			102.0
1959	19.7	44.6		82.2	100.1		113.2			138.1
Average	30.7	61.0		83.8	98.1		106.8			
N uptake (grain + stover (lb/acre))										
1956	80.0			125.0	128.0		149.0			174.0
1957	36.0			85.0	115.0		112.0			127.0
1958	50.0			87.0	109.0		135.0			137.0
1959	23.0			69.0	87.0		104.0			165.0
Average	47.3			91.5	109.8		125.0			150.8
Tifton, Ga (Sandy loam)										
Yield (Bushel - only grain)										
1958	64.9	83.4		103.5	112.5		108.5			
1959	48.8	66.3		77.3	80.2		85.1			
Average										
N uptake (grain + stover (lb/acre))										
1958	82.0			113.0	124.0		120.0			
1959	59.0			96.0	132.0		128.0			
Average	70.5			104.5	128.0		124.0			

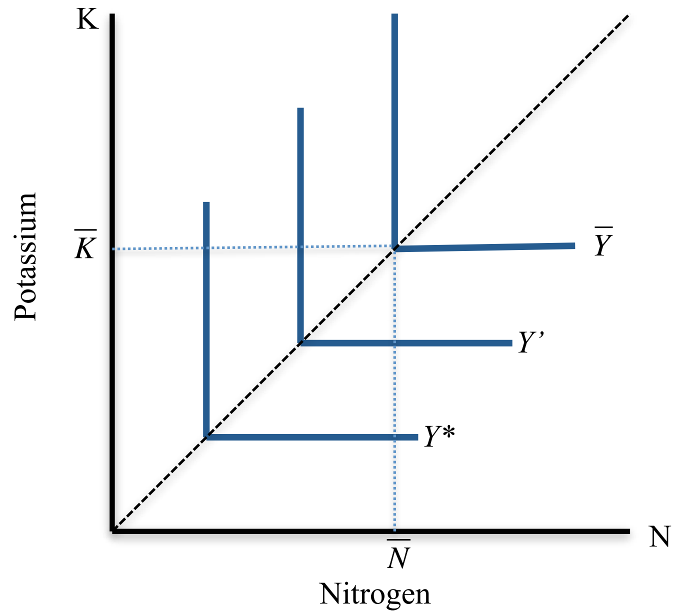


Figure 2.1a. Isoquants under Leontief technology

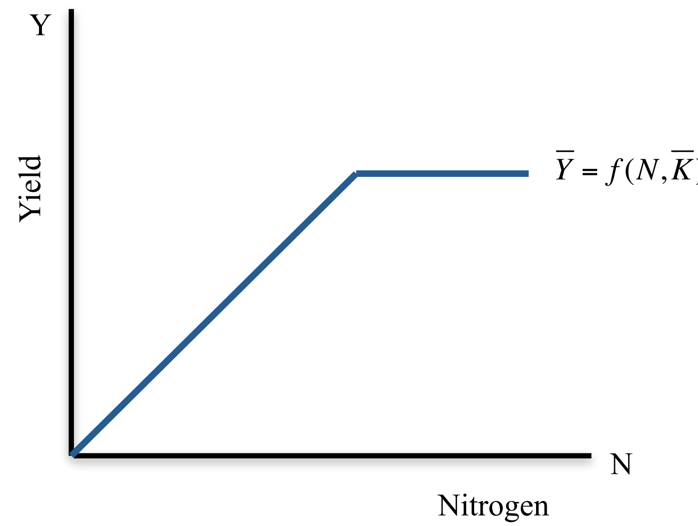


Figure 2.1b. Yield response under Leontief technology

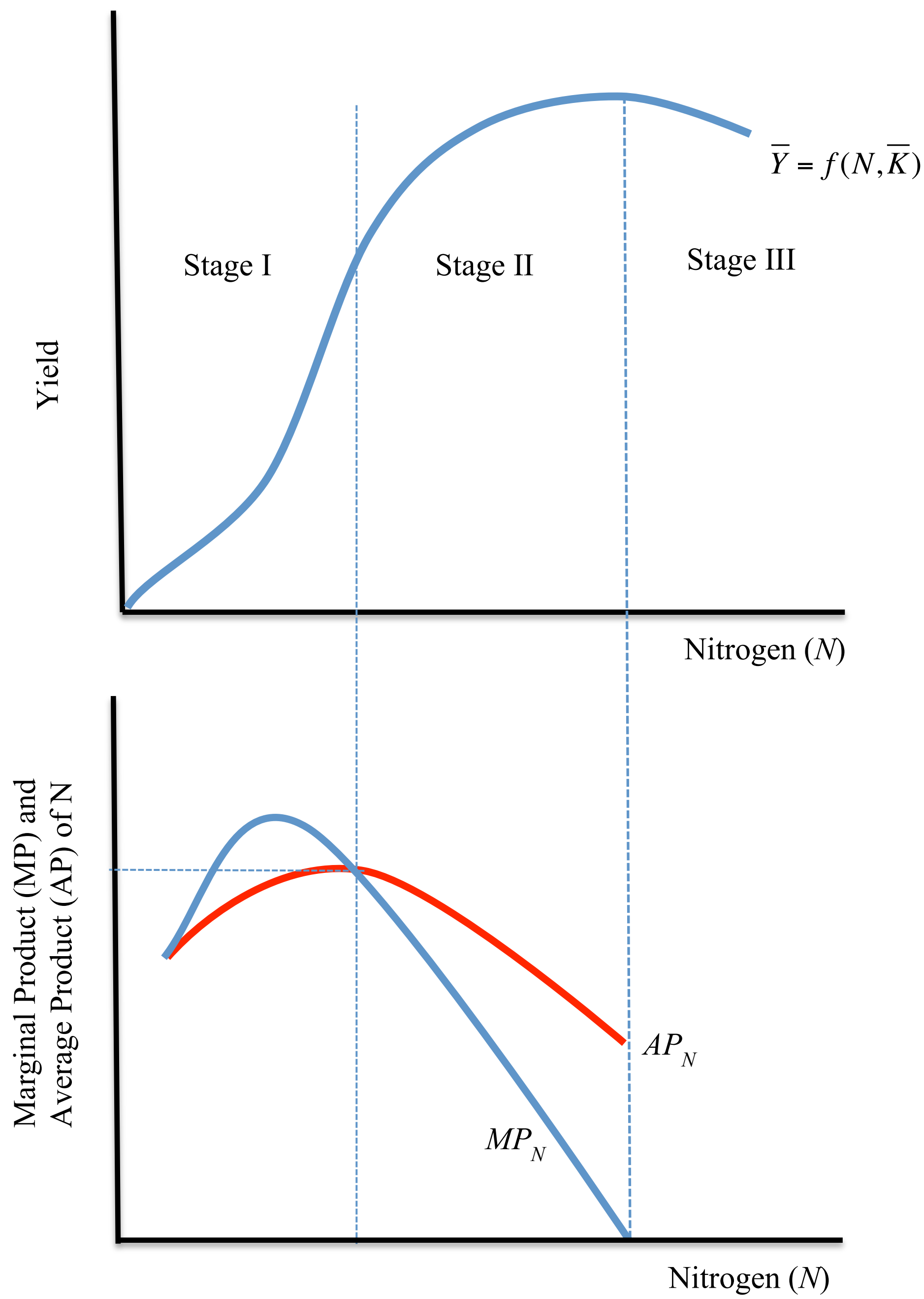


Figure 2.2 Each additional unit of *N* to the fixed input will increase total output by smaller and smaller increments.

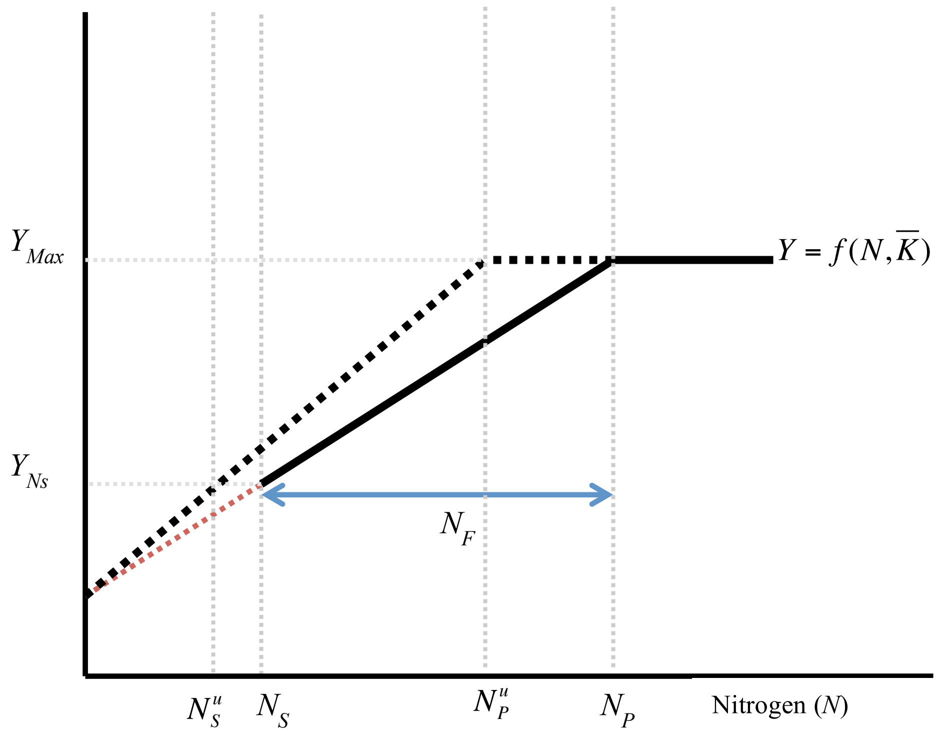


Figure 2.3. How to calculate the N fertilizer rate (N_F)

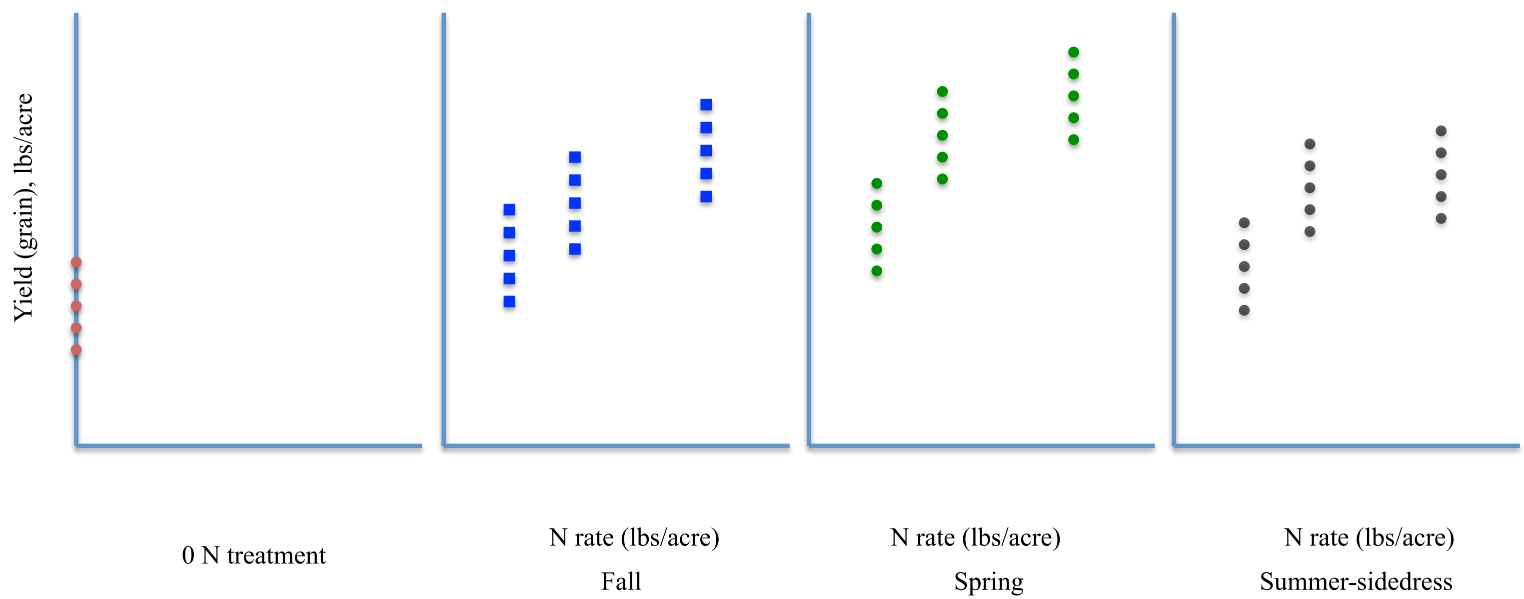


Figure 2.4. Illustration of raw data from one experiment

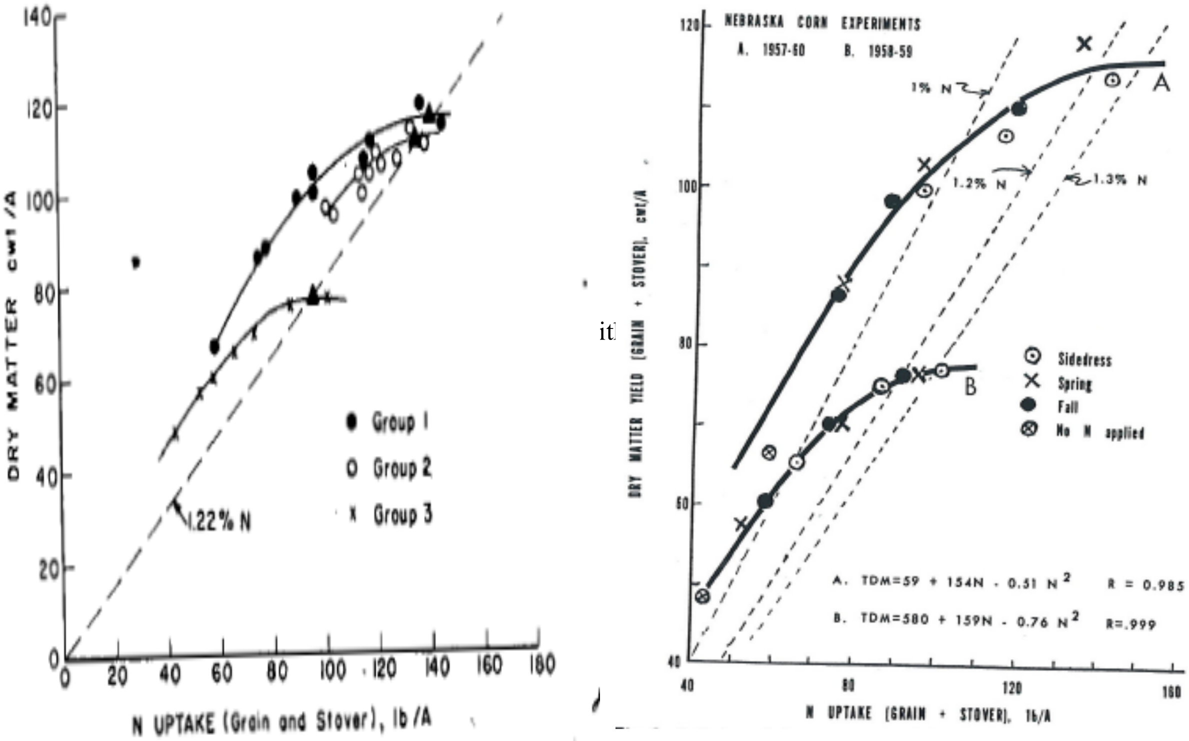


Figure 2.5. Relation of dry matter yield (corn grain plus stover), cwt/A, to total N uptake (grain plus stover), lbs/ac, for irrigated corn experiments in Nebraska, involving three application (fall, spring, and summer sidedress), three applied N rates (40, 80, and 160 lb N/acre), and a single zero-N treatment. Reprinted from “Nitrogen Requirements of Crops for Maximum Yield.” *In* W.H. McVickar et al. (ed.) *Agricultural Anhydrous Ammonia-Technology and Use,* by G. Stanford, 1966, Madison, WI: ACSESS-Alliance of Crop, Soil, and Environmental Science Societies. Copyright 1966 by ASA, CSSA, SSSA (left panel) and from “Rationale for Optimum Nitrogen Fertilization in Corn Production,” by G.Stanford, 1973, *Journal of Environmental Quality, no. 2,* p. 161. Copyright 1973 American Society of Agronomy (right panel). Reprinted with permission.

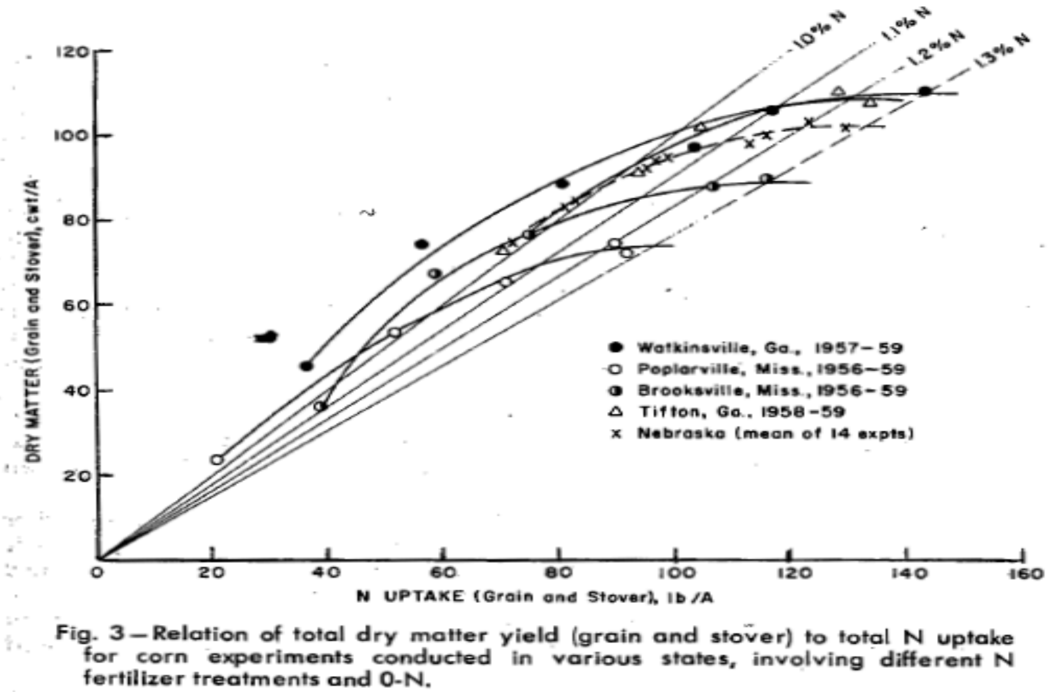


Figure 2.6. Relation of total dry matter yield to total N uptake at different locations. Reprinted from "Nitrogen Requirements of Crops for Maximum Yield." *In* W.H. McVickar et al. (ed.) *Agricultural Anhydrous Ammonia-Technology and Use,* by G. Stanford, 1966, Madison, WI: ACSESS-Alliance of Crop, Soil, and Environmental Science Societies. Copyright 1966 by ASA, CSSA, SSSA. Reprinted with permission.

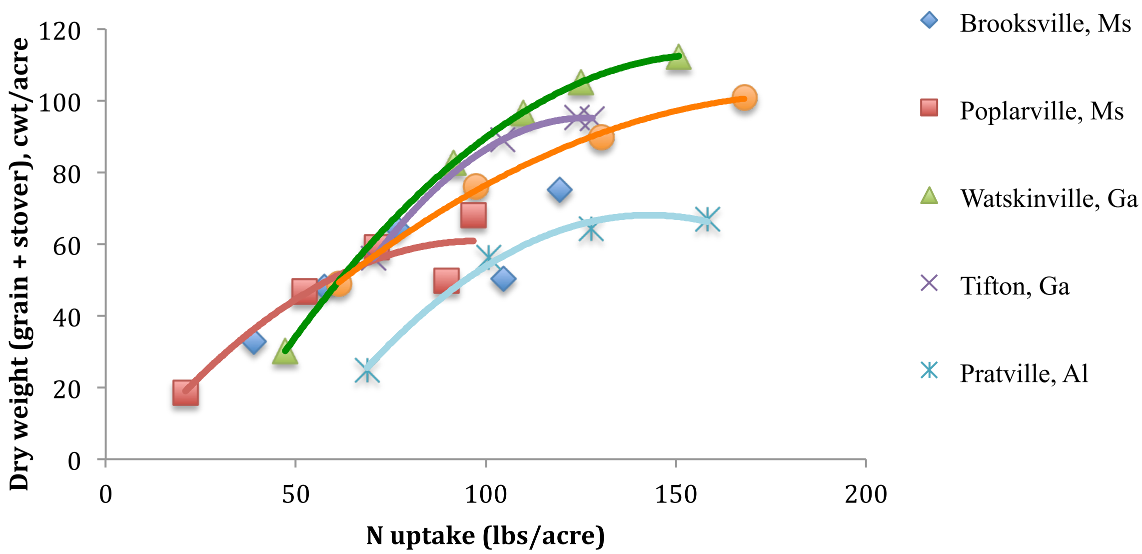


Figure 2.7. Replication of relation of total dry matter yield to total N uptake for corn experiments using Pearson et al. (1961) data.

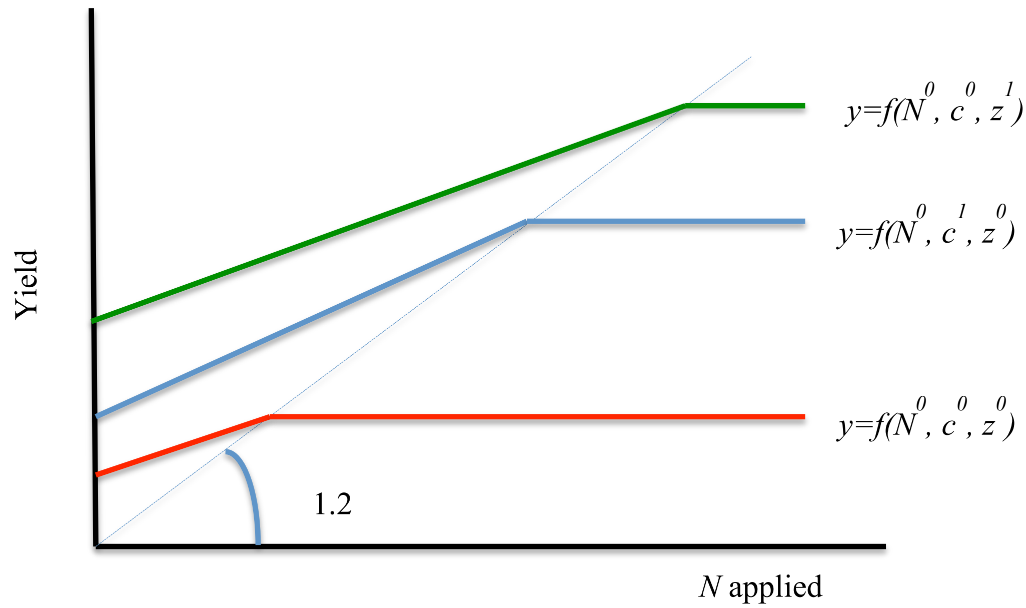


Figure 2.8. Kinks line up on a ray out of the origin with slope 1.2

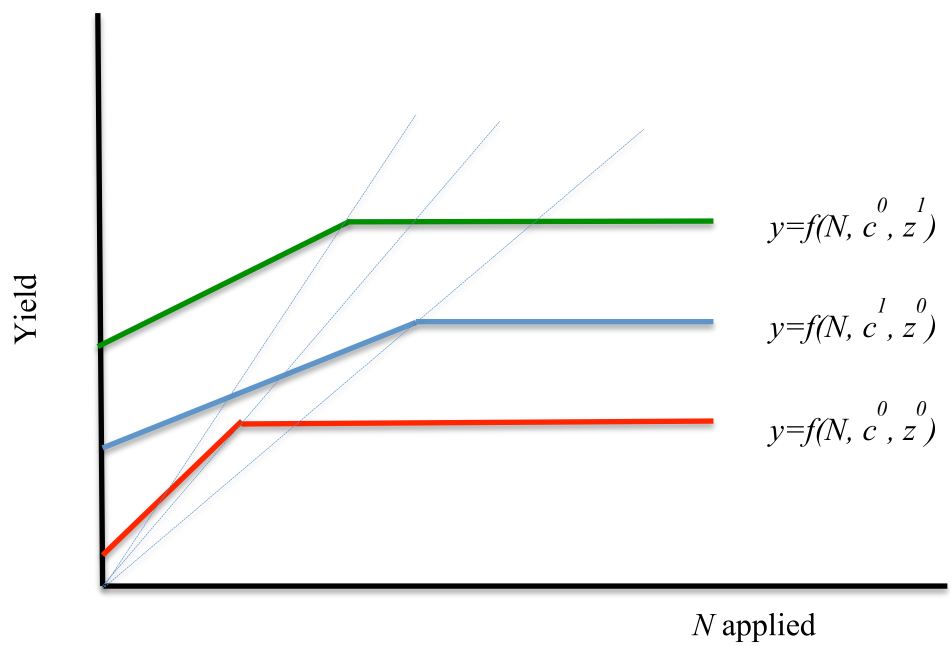


Figure 2.9. Kinks do not line up.

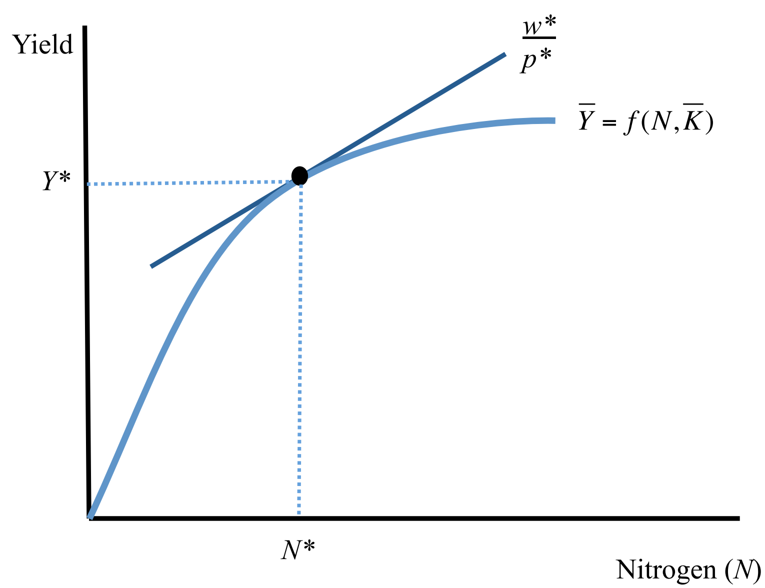


Figure 2.10a. Economically optimal N rate

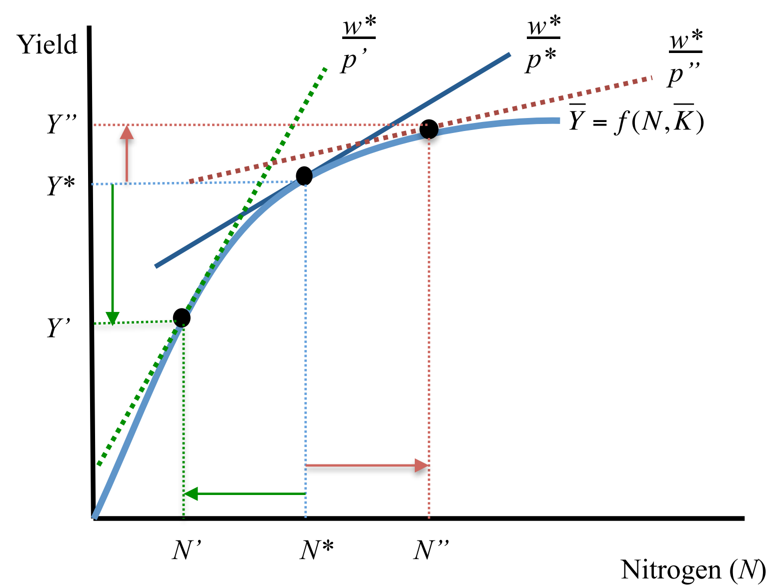


Figure 2.10b. Effects of relative price change on optimal N rate

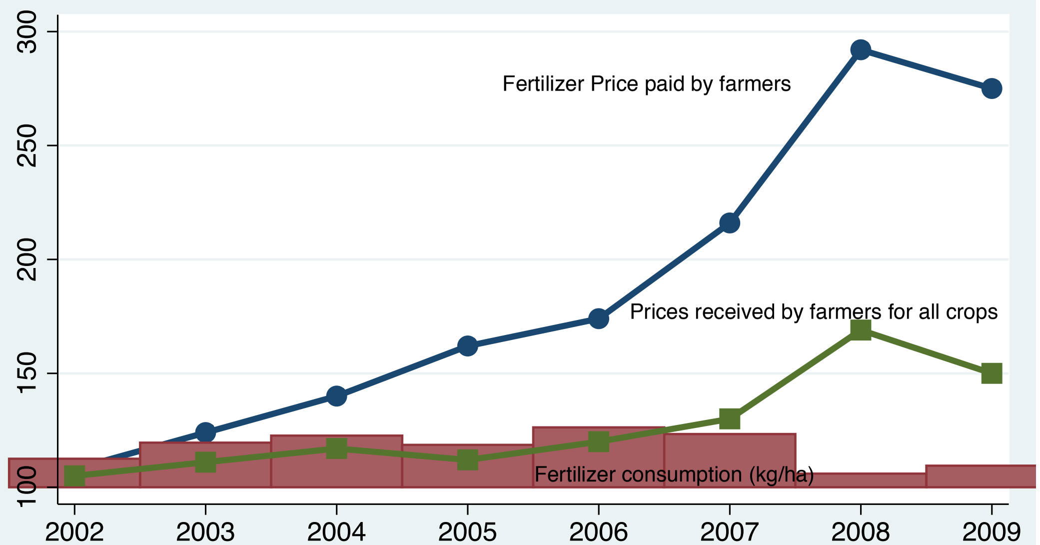


Figure 2.10c. Fertilizer Price and Consumption in the US, 2002-2009

Source: World Bank (2013)

Chapter 3

AN EMPIRICAL INVESTIGATION OF THE STANFORD'S 1.2 RULE OF FERTILIZER RECOMMENDATION

Most corn *N* recommendations in the United States have relied heavily on Stanford's 1.2 Rule. The use of "yield goal" or "yield potential" in formulating *N* fertilizer recommendations has been a common practice of corn farmers since the 1960s. Very recently, however, many land-grant universities have begun to admit that the use of Stanford's 1.2 Rule is faulty and inappropriate (Camberato, 2011). Extensive research has shown that expected, potential, average, or even actual yields are often very poorly correlated with the economically optimum *N* rate (Vanotti and Bundy, 1994a; Vanotti and Bundy, 1994b; Lory and Scharf, 2003). Most of the *N* recommendations for corn cost money in overapplied *N* fertilizer. These results clearly suggest that the use of yield goal as the primary management input is not the deciding factor in how much *N* the plant needs. Several states in the U.S. Corn Belt have abandoned the yield goal approach (e.g. Illinois) and moved toward data-driven recommendations.

There are several studies that pointed out problems in Stanford's 1.2 Rule. But most of these studies are based on the assumption that what Stanford concluded from his data was justified. In Chapter 2, I showed that Stanford's conclusions ignored several important statistical and empirical considerations in his study. I concluded that the1.2 Rule's long-term and widespread use basically resulted from its long-term and widespread use. But none of my conclusion implied that the rule is necessarily wrong.

Therefore, the objective of this study is to reexamine the validity of Stanford's 1.2 Rule as a yield-goal base factor for corn in the *N* rate recommendation system. As discussed previously, Stanford's approach in making *N* fertilizer recommendations makes economic sense only under very restrictive assumptions about the form of the yield response function. For the rule to be appropriate: (1) the response function must take on the von Liebig functional form; and (2) the kinks of the von Liebig response curves for different weather, soil type, and other factors of production must "line up" on a ray out of the origin with slope 1.2. During Stanford's time, most crop scientists have had little access to microeconomic theory and marginal analysis. Since Stanford did not consider prices or input substitution in his approach, the von Liebig functional form is implied. With von Liebig response function the EONR does not depend on input and output prices and marginal analysis. The EONR is the minimum rate at which yield reaches its plateau.

To test if Stanford's 1.2 Rule satisfies the above restrictions, I used non-linear estimation techniques and non-nested hypothesis framework. The data came from the long-term corn experiment from Illinois, Nebraska, and Iowa, in addition to the original datasets Stanford (1973) used in his analysis. My point was not to investigate whether von Liebig model is the appropriate one with which to estimate crop response, but to take the model as a starting point to test the validity of the 1.2 Rule. To my knowledge, this is the first and only paper that puts to a rigorous statistical and empirical test Stanford's conclusion -- i.e. the critical N concentration of the plant's dry matter is constant at 1.2 percent. Although several previous studies have cast doubt on the appropriateness of yield goal-based approaches, none have thoroughly investigated and confirmed Stanford's results. My analysis provides empirical evidence of whether Stanford's 1.2 Rule is indeed faulty, and whether the yield-goal approach to N fertilizer recommendation should be completely abandoned. In addition, my analysis offers insights into how N fertilizer recommendations could be better formulated. Important lessons can be learned from my study about the processes in which university-led farm management guidelines are developed, both in terms of flaws to be rectified and successes to be replicated. Although this study focuses on corn fertilizer recommendation, findings from this study may have broader implications for other crops, including rice and soybeans, whose *N* fertilizer requirements are also based on yield-goal based algorithm.

3.1. The von Liebig Crop Response Model

Stanford (1966, 1973) concluded that the validity of *N* fertilizer predictions depend largely on realistic estimates of yield, nitrogen fertilizer efficiency, and residual mineral *N* supply. He claimed that plant's *N* uptake will be vary around 1.2 times its dry matter yield, and that this result is consistent across a very wide range of growing conditions. Stanford's claim will only be valid if indeed the correct specification of the production function is von Liebig. Given this, I investigated the validity of Stanford's 1.2 Rule using the von Liebig function to model the crop response to *N*.

The von Liebig technology reflects the "Law of the Minimum" (Paris, 1992) whereas plant growth is constrained by the level of the scarcest nutrient, exhibiting zero elasticity of factor substitution. Mathematically, it can be expressed as (Chambers and Lichtenberg, 1996):

$$y = min\{f_1(x_1), f_2(x_2), \dots, f_n(x_n)\} + \mu \tag{3-1}$$

where $y \in R_+$ is the response variable (with R_+ denoting the set of nonnegative real numbers), $\boldsymbol{x} \in R$ is a vector of nutrients that affect crop growth, $x_i \in R_+$ represents the i^{th} element of $\boldsymbol{x}$, $f_n: R_+ \rightarrow R_+$ is an arbitrary real-valued function, and μ is the disturbance term. The minimum operator selects the level of yield that is associated with the limiting nutrient. With von Liebig functional forms, a plateau exists in which crop yield does not increase with the addition of non-limiting production inputs (APW, 1985). The prevalent form of the von Liebig model employed is the "linear response and plateau" model. The debate of the past century focused more around the linear specification of the von Liebig's hypothesis than about the idea of non-substitution and plateau.

Cate and Nelson (1971) were the first to estimate a linear plateau model. Anderson and Nelson (1975), along with Lanzer and Paris (1981) were pioneers in the use of linear response and plateau functions in agricultural economics. Lanzer and Paris (1981) suggested a method of estimating a plateau model using linear spline techniques. Following Anderson and Nelson (1975) and Lanzer and Paris (1981), there has arisen a large literature within agricultural production economics focused on devising appropriate methods for testing the von Liebig hypothesis. Ackello-Ogutu et al. (1985) pioneered the study of comparing the von Liebig model with polynomial models using non-nested hypothesis tests (Holloway and Paris, 2002.). Their nutrient non-substitution model follows Lanzer and Paris (1981) in their assumption of weak separability between soil and weather variables and fertilizer nutrients. Their study supported the findings of Anderson and Nelson (1975) that the use of polynomial approximations should be abandoned because it leads to costly biases whenever plateaus are significantly manifested.).

Grimm et al. (1987) also compared linear forms of von Liebig model with polynomial models using non-nested hypothesis tests on data on wheat, corn, cotton, silage, and sugar beets. Results suggested that the von Liebig model was more parsimonious as well as agronomically meaningful, and confirmed that polynomial models tend to overestimate the optimal input levels. Paris and Knapp (1989) extended the specification of the von Liebig model to include the random disturbances under the "*min*" operator in the model.

Frank et al. (1990) argued that the studies of Ackello-Ogutu et al. (1985), Grimm et al. (1987) and Paris and Knapp (1989) were limited to only comparing the von Liebig to polynomial specifications, but did not address the issues regarding the plateau growth and factor substitution. Frank et al. (1990), using Iowa data reported by Heady, Pesek, and Brown, extended the work of Ackello-Ogutu et al. (1985), Grimm et al. (1987) and Paris and Knapp (1989) by performing a series of non-nested tests to model corn yield response to nitrogen and phosphorus. They found out that the corn response is characterized by limited substitution between nitrogen and phosphorus and by a growth plateau. All the other alternatives were rejected in favor of the Mitscherlich-Baule model, including the linear response plateau specification of the von Liebig model. Cerrato and Blackmer (1990) published a similar paper comparing five crop response models but concluded that the quadratic-plus-plateau model best described the yield responses observed in their study.

The von Liebig's Law has also received critical re-evaluation from Berck and Helfand (1990) in an attempt to reconcile the von Liebig and quadratic functional forms. Berck and Helfand (1990) introduced the idea of linear plateau parameters varying stochastically. They expressed the von Liebig production function for two inputs as:

$$y = min(a_0 + a_1x_1 + \mu_1, b_0 + b_1x_2 + \mu_2, P + \mu_3) \quad (3\text{-}2),$$

where $\mu_j (j = 1,2,3)$ is the random error term. Unlike equation (3-1), equation (3-2) has separate error terms guiding each response function. They concluded that the quadratic form is still adequate for yield prediction goals, but that neither form appears to be better for estimating yield changes resulting from input level changes.

Using the same approach as Berck and Helfand (1990), Paris (1992) argued that the while the von Liebig hypothesis suggests non-substitution between nutrients and a yield plateau, it does not necessarily imply linearity of the relation between nutrients and yield. Since Frank et al. (1990) interpreted the von Liebig hypothesis in a linear framework, Paris argued that when the von Liebig hypothesis is interpreted in its more general framework using nonlinear regimes instead, the von Liebig might perform better than any other specifications. Paris (1992) demonstrated that equation (2) can be easily made consistent with the law of diminishing marginal productivities by choosing each response function to be concave. He argued that the potential yield functionscan either be linear or non-linear without danger of misspecifying the "direct relation" between nutrients and yield expressed by von Liebig. Assuming only two nutrients and Mitscherlich specification, a von Liebig function with additive errors can be stated as

$$y_i = \min\left\{m\left(1-k_N e^{b_N N_i}\right)+u_{Ni}, m\left(1-k_P e^{-b_P P_i}\right)+u_{Pi}\right\} \quad (3\text{-}3)$$

A linear von Liebig hypothesis can also be expressed as

$$y_i = \min\left\{a_N + b_N N_i + u_{Ni}, a_P + b_P P_i + u_{Pi}, m + u_{mi}\right\} \quad (3\text{-}4).$$

Though the functional forms in (3-3) and (3-4) involve the same nutrients, the number of parameters differ. Paris (1992) also assumed that the error associated with the dependent variable is unique, and hence it is not subject to the minimum operator. The nonlinear and linear von Liebig response models can be expressed as

$$y_i = min\{m(1-k_N e^{-b_N N_i}), m(1-k_P e^{-b_P P_i})\} + u_i \quad (3\text{-}5)$$

and

$$y_i = min\{\alpha_N + b_N N_i, \alpha_P + b_P P_i\} \quad (3\text{-}6),$$

respectively. Paris (1992) concluded that the von Liebig hypothesis interpreted nonlinearly is a more appropriate fit than polynomial specifications.

Chambers and Lichtenberg (1996), and subsequently, Berck, Geoghegan, and Stohs (2000), applied nonparametric methods to test the von Liebig hypothesis. Chambers and

Lichtenberg's (1996) analyses suggested the existence of input substitution and yield plateaus. Berck, Geoghegan, and Stohs (2000) disagreed, citing little evidence for right-angle isoquants. Llewelyn and Featherstone (1997), using a synthetic or engineering approach to evaluate the same functional forms considered by Paris (1992)[17], found evidence for both Mitscherlich-Baule formulation and a non-linear von Liebig which corroborated Paris's findings.

In 2002, Holloway and Paris attempted to estimate a frontier von Liebig crop response model using Bayesian techniques. They consider a model where both experimental error and the inefficiency term enter additively outside the minimum operator as follows:

$$y = min(a_0 + a_1 x_1, b_0 + b_1 x_2, P) + \mu + \varepsilon \qquad (3\text{-}7),$$

where ε is the "inefficiency term". Akin to the specifications of Paris and Knapp (1989), Berck and Helfand (1990), and Paris (1992), Tembo et al. (2003, 2008) suggested an alternative specification of the von Liebig production function:

$$y = min\{f_1(x_{1jt}, \beta_1), \dots, f_n(x_{njt}, \beta_n), P_t\} + \mu_{jt} \qquad (3\text{-}8),$$

where subscripts *t* and *j* index the year and the cross-sectional unit for each factor *i*.

3.2. Data and Estimation

3.2.1. Data

This study used the published data in Pearson et al (1961). These experimental results were the original data set that Stanford (1973) used in the analysis of his paper. Complete details of the data can be found in Chapter 1. In addition to these data sets, I also analyzed data from long-term experiments in Illinois, Nebraska, and Iowa (Table 3.1). The data set contains information on corn grain yields, dry matter yield, *N* fertilizer application rates, and *N* uptake.

[17] They used the CERES-Maize simulator to produce yield data to evaluate production functions that take into account climatic and soil conditions as well as N and irrigation inputs.

The experimental data from Illinois and Iowa, however, only contain information on corn grain yield and *N* fertilizer application rates. The Iowa data came from the earlier works of Binford, Blackmer, and Cerrato (1992) and Blackmer et al. (1989) in 15 experimental locations across the state between 1985 and 1990 (N=1998). Nitrogen fertilizer rates ranged from 0 to 300 pounds per acre in 25-50 pound increments, with three repetitions of each application rate performed annually at each experiment station site. The Illinois data, on the other hand, came experimental plots in Monmouth and Perry conducted from 1980 to 2012 (n=720). Nitrogen fertilizer rates ranged from 0 to 320 pounds per acre in 20-60 pound increments, with three repetitions of each application rate performed annually at both locations. There were two sets of data from Nebraska. The first set of experimental data, conducted from 1969 to 1983, was from the Nebraska Agricultural Experiment Station Field Laboratory near Mead, NE, which was used in the previous work of Olson et al., (1986).[18] A split-block factorial design repeated over time was employed, with two whole blocks, each with four randomized blocks. Nitrogen fertilizers were applied at 90, 180, and 270 pounds per acre and two check plots outside of the factorial were included in each replication. The second set of experimental data, which contains 1383 observations, came from 17 experimental locations representing the main corn production areas of Nebraska including Mead from 2002 to 2004 (Dobermann et al., 2011). The *N* rates applied ranged from 0 to 300 pounds per acre. Individual plots were arranged in a randomized complete block design with four replications at each site.

[18] Note that this is one of the experimental sites in Stanford's 1966 and 1973 paper. Olson's data that were used by Stanford (1973) for his analysis were no longer available.

3.2.2. Estimation Strategies

3.2.2.1. Estimation of the von Liebig Model

Using Paris's (1992) approach, the von Liebig formulation with linear potential yield function can be expressed as:

$$y_{it} = min\{\theta_0 + \theta_N N_{it}, P\} + u_{it} \quad (3\text{-}9)$$

where y_{it} is the dry matter yield (pounds per acre) in the *i*th plot at time *t*, N_{it} is the level *N* uptake (pounds per acre) as the limiting input, $u_{it} \sim N(0, \sigma_e^2)$ is the disturbance term, P is the maximum or plateau yield, and θ_0 and θ_N are the parameters of the model.

Since there were no available record on the dry matter yield and *N* uptake of plant's dry matter from the experimental field plots in Illinois and Iowa, I used the grain yield and the amount of *N* fertilizer applied in these states. Note that I assumed the error associated with the dependent variable is unique and therefore, not subject to the minimum operator. Stanford simplified the problem by assuming that year effect and temporal variability (i.e. rainfall, temperature, relative humidity, among others) can be ignored completely. The plateau is also assumed to be nonrandom, in spite of its determinants being stochastic (APW, 1985; Paris and Knapp 1989; LLewelyn and Featherstone, 1997). This assumption suggests that all factors that define the plateau are fixed and completely controllable.

Equation (3-9) is an example of a non-linear regression function, $m(\mathbf{x},\theta), \theta \in \Re^P$, where m is a known function of $\mathbf{x}$, a *K*-vector, and θ, a $P \times 1$ parameter vector. The standard non-linear regression model can be defined by

$$y = m(\mathbf{x},\theta_0) + u \quad (3\text{-}10),$$

where u are scalar i.i.d. random variables with $E(u \mid \mathbf{x}) = 0$ and σ_0^2. Unlike the linear model where $f(x, \beta_0) = x'\beta_0$, the dimensions of the vectors x and β_0 are not necessarily the same (Amemiya, 1983).

Equation (3-10) can be estimated directly by maximizing its corresponding likelihood function (Paris and Knapp, 1989; Paris, 1992). However, the maximum likelihood method requires one critical assumption, i.e. the true Data Generating Process (DGP) is known to lie within a specified probability distribution. This is to say that the model of the given data is correctly specified. If the correct distribution is something other than what is assumed, then the likelihood function is misspecified and the desirable properties of the maximum likelihood estimator (MLE) might not hold.[19] The most common probability distribution assumed when doing the maximum likelihood estimation is the normal distribution. The normal MLE is quasi-maximum likelihood and produces consistent estimates if the mean is correctly specified.[20]

Given this, I estimated the von Liebig model using nonlinear least squares. The idea behind this method is that it finds the non-linear least squares (NLLS) estimator, denoted by $\hat{\theta}$, which is defined as the value of θ that minimizes the sum of squared residuals between y and $m(\mathbf{x}, \theta)$. That is, $\hat{\theta}$ solves

$$\min_{\theta \in \Theta} N^{-1} \sum_{i=1}^{N} \left[y_i - m(\mathbf{x}_i, \theta) \right]^2 \qquad (3\text{-}11).$$

Note that the θ appearing in equation (3-11) is an argument of the function $m(\mathbf{x}, \cdot)$ and θ_0 in equation (3-10) is a fixed true value. One only needs to supply the function, $m(\mathbf{x}, \theta)$, in this

[19] The MLE is most attractive because of its large sample properties.
[20] The idea of quasi-maximum likelihood is that there is a family of densities whose first order condition (the score) with respect to the parameters in the mean is exactly the same. Such a family of distribution is called the exponential family or exponential models (Wooldridge, 2010).

case, equation (3-9). To begin the process, I provided initial values for the parameters. Finding the starting values for a nonlinear procedure can be difficult. I used the parameter estimates from the quadratic production function to determine the maximum possible yield, *P,* and nutrient uptake and then used these values as starting points for the nonlinear procedure.

Given the parameter estimates from the linear von Liebig functional form, the critical *N* concentration of plant's dry matter, denoted by θ, can be derived by dividing the height of the plateau, *P*, by the minimum *N* required to achieve *P*, (N^k). That is,

$$\theta = \frac{P}{N^k} = \frac{P\theta_N}{P-\theta_0} \tag{3-12),}$$

where

$$N^k = \frac{P-\theta_0}{\theta_N} \tag{3-13).}$$

I then tested

$$H_0 : \theta = 1.2 \tag{3-14).}$$

If H_0 can be rejected, then Stanford's 1.2 Rule misleads.

3.2.2.2. Do kinks of the von Liebig response curve line up on a common ray?

To see whether the kinks all line up on a common ray out of the origin, suppose there are:

$$\begin{aligned} y_1 &= m(x_1,\theta_1)+u_1 \\ y_2 &= m(x_2,\theta_2)+u_2 \\ &\vdots \\ &\vdots \\ y_G &= m(x_G,\theta_G)+u_G \end{aligned} \tag{3-15}$$

where *G* stands for locations or states and $E\left[u_g \mid \mathbf{x}\right]=0, g=1,2,...,G$. I tested the null hypothesis:

$$Ho : \theta_1 = \theta_2 = ... = \theta_G \tag{3-16}$$

where θ_1 is the calculated critical N concentration of plant's dry matter for state 1, θ_2 is the critical N concentration for state 2, and θ_G is the critical N concentration for state G. I jointly estimated the equations in (3-15) described by a nonlinear equation system. The parameters were estimated by applying the nonlinear seemingly unrelated regression to the system of equations. I tested the null hypothesis about the estimated parameters from the fitted model. If the null hypothesis is rejected, it suggests that the critical N concentration of plant's dry matter is not constant at 1.2 and the Stanford's 1.2 Rule basis of fertilizer recommendation is inappropriate.

3.2.2.3. Is von Liebig the correct functional form?

To determine whether the von Liebig model is the correct model specification, I used a non-nested hypothesis framework as proposed by Davidson and MacKinnon (1982). I tested the equation

$$H_0 : y_i = m\left(x_i, \theta\right) + u_{oi} \qquad \text{(3-17)},$$

where u_{oi} is assumed to be $N\left(0, \sigma_0^2\right)$. Suppose an alternative hypothesis is plausible:

$$H_1 : y_i = g\left(z_i, \gamma\right) + u_{1i} \qquad \text{(3-18)},$$

where z_i is a vector of observations on exogenous variables, γ is the vector of parameters to be estimated and u_{1i} is $N\left(0, \sigma_1^2\right)$ if H_1 is true. For the purposes of this study, I tested three alternative hypotheses: two polynomial specifications (the quadratic and the square-root functions) and the Mitscherlich-Baule specifications. The quadratic model is defined by

$$y_i = \gamma_0 + \gamma_1 N + \gamma_n N^2 + u \qquad \text{(3-19)},$$

where y_i is the grain yield (bushels/acre) or the total dry matter weight (pounds/acre) and N is the rate of N application (pounds/acre) or N uptake (pounds/acre) and γ 's are the parameters to be estimated. The square root model is defined by

$$y_i = \gamma_0 + \gamma_1 N + \gamma_2 N^{1/2} + u \qquad (3\text{-}20),$$

while the Mitscherlich-Baule model is defined by

$$y_i = P\left(1 - ke^{-\gamma N_i}\right) + u \qquad (3\text{-}21).$$

Following Davidson and MacKinnon (1982), the form of the compound model can be expressed as:

$$y_i = (1-\alpha)m(x_i,\theta) + \alpha g(z_i,\gamma) + u_i \qquad (3\text{-}22).$$

Simplifying equation (3-22),

$$y_i = m(\mathbf{x},\theta) + \alpha\left[g(\mathbf{z},\gamma) - m(\mathbf{x},\theta)\right] + u_i \qquad (3\text{-}23).$$

If $\alpha = 0$, then H_0 is the correct model and if $\alpha = 1$, then it implies H_1. In principle, H_0 could be tested by testing $\alpha = 0$. It is impossible, however, to estimate α, θ, and γ jointly. Davidson and MacKinnon (1983) suggested that a simple solution would be to replace γ by its predicted value, $\hat{\gamma}$, under H_1. The composite model becomes

$$y_i = (1-\alpha)m(x,\theta) + \alpha\hat{g} + u_i \qquad (3\text{-}24).$$

A test of $\alpha = 0$ is known as *J*-test and is a routine t-test.

Since the H_0 involves a nonlinear model, equation (3-24) is also a nonlinear regression, and one which may be computationally difficult if H_0 and H_1 are very similar. To overcome this problem, equation (3-24) can be linearized around the point $\alpha = 0$ and $\theta = \hat{\theta}$, so as to obtain the linear regression

$$y - \hat{m} = \hat{\mathbf{M}}\theta + \alpha\left(\hat{g} - \hat{m}\right) + u \tag{3-25}$$

where $\hat{m} = m\left(x_i, \hat{\theta}\right)$, $\hat{g}_j = g_j\left(z_j, \hat{\gamma}\right)$ and $\hat{\mathbf{M}}$ is the matrix of derivatives of m with respect to θ, evaluated at the non-linear square estimates $\hat{\theta}$. This procedure is called a *P* test. If the null hypothesis that $\alpha = 0$ is not rejected, then von Liebig model is the correct model specification.

The *P* test can be easily extended to handle several alternative hypotheses. Let the null hypothesis still be H_0, given by equation (3-10), and the alternative hypotheses be

$$H_j : y = g_j\left(z_j, \gamma_j\right) + u_j, \quad j = 1, \ldots, J. \qquad \text{(3-26).}$$

The compound model becomes

$$H_c : \left(1 - \sum_{j=1}^{J} \alpha_j\right) m\left(x, \theta\right) + \sum_{j=1}^{J} \alpha_j g\left(z_j, \gamma_j\right) + u \qquad \text{(3-27),}$$

and the corresponding *P* test regression is

$$y_i - \hat{m} = \sum_{j=1}^{j} \alpha_j\left(\hat{g}_j - \hat{m}\right) + \hat{\mathbf{M}}\theta + u_i \qquad \text{(3-28).}$$

The appropriate test statistic is then an asymptotic *F* test of the hypothesis that $\alpha_1 = \alpha_2 = \ldots = \alpha_j = 0$.

3.3. RESULTS AND DISCUSSION

The summary statistics for all the explanatory variables in the non-linear estimation that are used throughout the study are presented in Tables 3.2. In this section, I presented formal statistical and empirical evidence about whether the two restrictions mentioned above were satisfied.

3.3.1. Stanford's 1.2 Rule: a case of overgeneralization

Table 3.3 presents the estimation results using the von Liebig model by U.S. state. Except for the intercept, θ_0, in Georgia in section A, all the parameters were found to be significant at the 1 percent level, indicating a clear response for corn to applied *N*. The values of θ, which represented the critical *N* concentration of corn yield in each U.S. state, ranged from 0.62-0.86 suggesting that the maximum yield is achieved at no more than 1.2 percent *N* concentration. The hypothesis, $H_0 : \theta = 1.2$, was rejected in *F* tests for each state (in AL, $F(1,77) = 140.19$, *p-value* = 0.00; GA, $F(1,45) = 54.42$, *p-value* = 0.00; MS, $F(1,58) = 359.41$, *p-value* = 0.00; NE, $F(1,1630) = 3158.06$, *p-value* = 0.00). The maximum attainable yield was not associated with 1.2 percent *N* concentration in total dry matter. Given the parameter estimates in section A, fertilizer recommendations based on the 1.2 Rule overestimated the minimum *N* requirement of corn, (N^k), necessary to achieve maximum yield potential and hence fertilizer recommendations given to farmers result to over-fertilization. In Alabama for example, the estimated θ was 0.62 of plant's dry matter. Since Stanford assumed that corn typically has a harvest index of 50 percent and a bushel of shell corn contains 49.3 pounds dry matter, making total above ground dry matter 98.6 pounds (grain plus stover), a corn plant only needs to absorb 0.61 pounds of *N* to achieve one bushel of corn (98.6 x 0.62%) with adjustments on other factors, and not 1.2 pounds of *N*.

When using the estimates in Section B, where grain yield and *N* rate applied were used in the estimation of the linear von Liebig production function, a farmer in Alabama needs to apply about 0.63 pounds *N* per bushel of corn instead of 1.2 pounds.[21] If the yield goal is set at estimated *P*=80.39 bushels per acre (which is assumed to have 12% water), then its equivalent dry matter is equal to 70.74 bushels/acre grain and there is about 70.74 bushels/acre of stover.

[21] A bushel of corn is assumed to be 56 pounds. The total above ground yield on a per bushel basis is 112 pounds.

The total dry matter is then equal to 141.48 bushels/acre. A fertilizer recommendation using a factor of 1.2 pounds *N* per bushel of expected yield would have predicted a fertilizer need of about 170 pounds of *N* per acre or 81 pounds *N* per acre in excess of the predicted *N* based on this analysis with adjustments for fertilizer efficiency and existing nutrients in the soil. The excessive *N* use due to Stanford's 1.2 Rule was not acceptable from either an economic or environmental viewpoint. The farmer would decrease his profit by $34 per acre if the 1.2 Rule was followed. This would vary with *N* and corn prices. The cost presented here was based on *N* costing 42 cents per pound and corn price at $5 per bushel. The excessive *N* use was also a potential pollution hazard as fertilizer *N* application in excess of crop need dramatically increases residual *N* in the soil, which is likely to move into the ground or surface waters (Olsen et al., 1970; Lory, et al., 1995). Note, however, that there are cases when the Stanford's 1.2 Rule is correct and can also result to under-fertilization. I failed to reject the hypothesis, $H_0 : \theta = 1.2$, in Georgia ($F(1,45) = 0.96$, *p-value* = 0.3322), Iowa ($F(1,1995)$, *p-value* = 0.5522) and Illinois ($F(1,717)$, *p-value* = 0.7795). If indeed the correct functional form is linear von Liebig, then Stanford's 1.2 Rule does not mislead in these states. On the other hand, I rejected the hypothesis in Nebraska ($F(1,1630) = 80.83$, *p-value* = 0.00). This suggests that the 1.2 factor in Stanford's rule needs to be adjusted given the correct functional form is indeed linear von Liebig.

While Stanford (1973) claimed that the critical *N* concentration of the plant's dry matter is constant, results also showed that θ across U.S. states took on different values and were statistically different from each other (Table 3.4). The tests were performed by temporarily holding each θ in each state as null and testing in a pair-wise fashion with θ from a different state and against all the other θs. The results implied that the kinks of the linear von Liebig response curves did not line up on a common ray. For example, the hypothesis that $\theta_{AL} = \theta_{GA} = \theta_{MS} = \theta_{NE}$

was rejected (*chi-square*(3) = 99.98, *p-value* = 0.00). This suggests that the critical concentration for corn (on *N*) varies in every state. This was also evident when the predicted values of dry matter yield were plotted against the *N* uptake (Figure 3.1). Although the kinks seemed to be quite close to each other especially those from AL, GA, and MS, they did not line up on a common ray.[22] The results did not corroborate with Stanford's findings. The estimated optimum *N* rate needed in each specific state would be different. On the other hand, when data on grain yield and *N* applied were used, the critical concentration of *N* is similar in GA and IA, GA and NE, and IA and IL (Table 3.4 and Figure 3.2).

I also tested Stanford's 1.2 Rule at the experimental station level and in general, results suggested that θs were statistically different from 1.2 (Tables 3.6-3.11) and kinks did not line up on a common ray (Table 3.12). Extension agents' fertilizer recommendations given to farmers could not be based on state recommendation since fertilizer recommendations would differ in every site given different θs. For example in Mississippi, based on the estimated parameters the amount of *N* required per bushel of corn is 0.71 pounds in Brooksville and 0.78 pounds in Poplarville and were statistically different from each site at 1% level. However, extension agents could use the same fertilizer recommendations for sites in Georgia and Illinois in this study because θs were not statistically different from each other given the correct functional is linear von Liebig. *In general, all results suggest that the critical concentration of N varies across and even within fields, and hence site-specificity matters in making fertilizer recommendations.*

[22] Nebraska appears to be much different than the other states in terms of yield plateau because the state has the highest irrigated land per country. The water supply comes from the Ogallala Aquifer and reservoirs that capture water from snow-melt and rains.

3.3.2. What is the correct functional form?

The non-nested hypothesis results based on a *P* test are reported in Table 3.5. The tests were performed by temporarily holding each hypothesis as null and testing in a pair-wise fashion with each alternative and against all alternatives. The quadratic functional form outperformed all the rival specifications, both on a pairwise comparison as well as in a collective test against all alternatives in Illinois. The results also indicated that the Mitscherlich-Baule model is more appropriate than any other alternatives in Nebraska. As for the other states, the results were inconclusive. In Alabama the null hypothesis that $\alpha = 0$ rejected at 10 percent level suggested that the von Liebig model is not the correct specification when it is the null hypothesis. Information is insufficient however to choose the correct model specification from among the alternative models. Neither the polynomial functions nor the Mitscherlich-Baule function was rejected over any other model when they were the null hypothesis. In Iowa, the non-nested hypothesis test rejected square root and linear von Liebig functions but failed to reject quadratic and Mitscherlich-Baule functions. Failure to reject these alternatives in Alabama and Iowa suggested that the response between yield and *N* fertilizer tended to be smooth and allowed diminishing marginal productivity. If this is the case, then the marginal product schedule and the input and output prices matter in the determination of the EONR. Given a non-zero price ratio, there is a difference between the yield maximizing and profit maximizing input levels. An optimum fertilization level is attained when the marginal product of fertilizer is equal to the fertilizer price and output price ratio. In Georgia and Mississippi, none of the four specifications was rejected. The data did not allow us to say much about which functional form best represented corn response to *N* fertilizer.

Out of the 42 experimental locations in the study, only in two locations that the linear von Liebig model outperformed all the rival specifications, both on a pairwise comparison as well as in a collective test against all alternatives (Tables 3.13-3.18). The non-nested hypothesis tests rejected the linear von Liebig but failed to reject quadratic, square-root, and Mitscherlich-Baule in other 18 locations. In all the remaining experimental locations, the non-nested hypothesis tests favored none of the four rival specifications. The results were inconclusive on what the best specification to interpret the data set.

3.4. SUMMARY AND CONCLUSION

For a long time the Stanford's 1.2 Rule has provided general guidelines for *N* fertilizer management. With the 1.2 Rule, farmers could seemingly determine on their own, if only very roughly, their crops' fertilizer rate requirements using only the yield potential of their fields and the amount of N in the soil. However, there were numerous questionable economic and statistical procedures in the formulation of the 1.2 Rule due to Stanford's lack of access to modern statistical analysis and microeconomic principles. For the rule to make economic sense, the corn production technology must satisfy two restrictions: (1) the production function is von Liebig; and (2) the kinks of the von Liebig response curves for different weather, soil type, and other factors of production line up on a ray out of the origin with slope 1.2. Using nonlinear estimation techniques and non-nested hypothesis framework, I tested if the 1.2 Rule met these restrictions. Testing the validity of the 1.2 Rule is important to decide whether this approach to *N* fertilizer recommendation should be completely abandoned or followed.

I provided no empirical evidence in the study that supports Stanford's 1.2 Rule. The linear von Liebig production function was rejected in various locations and the kinks of the von

Liebig response curves for different growing conditions did not lie on a ray out of the origin with slope 1.2. The production function and the critical concentration of *N* can vary widely, both among states, and within states and therefore the level of optimal *N* also varies. Site-specificity matters in making fertilizer recommendations.

Stanford's 1.2 Rule results to either under- or over-application of fertilizer, and the economic analysis indicates that the consequences of using the 1.2 Rule can be large. In cases when the critical *N* concentration is indeed 1.2, the linear von Liebig production function is not the correct functional form, and hence the 1.2 Rule can be misleading. It is noteworthy to revisit the fertilizer recommendation procedures that rely on the 1.2 Rule and test if they satisfy the two restrictions mentioned above to ensure that *N* fertilizer is applied optimally and is readily available during crop growth periods. Unlike before, data from high-quality agronomic experiments and the necessary statistical and empirical procedures for such an empirical test are now available. This is an area of research in great need of interdisciplinary research among agronomists and agricultural economists.

Table 3.1. Description of experiment duration and N fertilizer rate, all sites

Location	Year of Experiment	Nitrogen Fertilizer Rate
Alabama		
Belle Mina	1955, 1957, 1959	0, 50, 100, 200
Pratville	1955, 1957, 1960	0, 50, 100, 201
Thorsby	1956, 1958, 1959	0, 50, 100, 202
Georgia		
Tifton	1958, 1959	0, 30, 60, 90, 120
Watskinville	1957, 1958, 1959	0, 30, 60, 90, 121
IA		
Site0	1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site1	1986-1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site3	1986-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site4	1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site5	1985-1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site6	1985-1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site8	1986-1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site9	1986-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site10	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site11	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site12	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site13	1987-1989	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site14	1989	0, 50, 100, 150, 200, 250, 300
Site15	1989	0, 50, 100, 150, 200, 250, 300
Site16	1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site17	1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Illinois		
Monmouth	1983-2012	0, 60, 120, 180, 240
Perry	1980-1992	0, 60, 80, 120, 160, 180, 240, 320
Mississippi		
Brooksville	1956, 1957, 1959	0, 50, 75, 100, 150, 200
Poplarville	1956-1959	0, 50, 75, 100, 150, 200

Table 3.1 Continued…

Location	Year of Experiment	Nitrogen Fertilizer Rate
Bellwood	2002-2003	87, 105, 112, 145, 162, 185, 187, 212, 235, 287, 335
Box Butte	2002-2004	0, 75, 100, 125, 150, 175, 200, 223, 300
Brosius	2004	0, 100, 150, 171, 200, 300
Brunswick	2002-2004	0, 50, 75, 100, 125, 150, 170, 175, 250
Cairo	2002-2004	0, 92, 100, 125, 132, 150, 156, 175, 192, 200, 210, 225, 300
Clay Center	2002	0, 92, 125, 175, 225, 300
Concord	2002-2004	0, 50, 75, 100, 110, 125, 150, 175, 250
Funk	2004	0, 100, 150, 200, 207, 300
Mead	1969-2004	0, 50, 75, 90, 100, 119, 125, 131, 140, 150, 175, 180, 250, 270
N. Platte	2002-2003	0, 100, 125, 150, 175, 180, 195, 200, 225, 300
North Bend	2004	0, 50, 100, 110, 150, 250
Paxton	2002-2003	0, 100, 125, 150, 175, 200, 212, 214, 225, 300
Pickrell	2003-2004	0, 50, 100, 123, 131, 150, 250
Scal	2002-2004	0, 50, 75, 100, 115, 125, 150, 175, 250
Scottsbluff	2002-2003	0, 75, 100, 125, 150, 175, 200, 225, 300
Spurgin	2004	0, 100, 150, 193, 200, 300
Wymore	2002	0, 75, 112, 125, 175, 250

Table 3.2. Descriptive statistics

Variable	No.of observation	Mean	S.D.	Min	Max
Alabama					
Grain yield (bu/acre)	72	69.49	18.83	18.50	108.90
Dry matter yield (grain + stover, cwt/acre)	72	68.49	18.56	18.23	107.33
Nitrogen applied (lbs/acre)	72	94.44	50.04	0.00	200.00
Nitrogen uptake (grain + stover, lbs/acre)	72	106.44	29.04	46.00	202.00
Georgia					
Grain yield (bu/acre)	43	75.83	21.00	19.70	113.20
Dry matter yield (grain + stover, cwt/acre)	43	74.73	20.70	19.42	111.57
Nitrogen applied (lbs/acre)	43	82.33	34.70	0.00	120.00
Nitrogen uptake (grain + stover, lbs/acre)	43	87.02	26.83	23.00	149.00
Iowa					
Grain yield (bu/acre)	1998	127.66	45.32	4.12	218.08
Nitrogen applied (lbs/acre)	1998	127.93	93.45	0.00	300.00
Illinois					
Grain yield (bu/acre)	720	122.01	56.58	0.40	217.47
Nitrogen applied (lbs/acre)	720	136.67	99.68	0.00	320.00
Mississippi					
Grain yield (bu/acre)	58	49.94	22.53	6.70	90.30
Dry matter yield (grain + stover, cwt/acre)	58	49.22	22.21	6.60	89.00
Nitrogen applied (lbs/acre)	58	94.83	55.16	0.00	200.00
Nitrogen uptake (grain + stover, lbs/acre)	58	62.81	31.81	9.00	153.00
Nebraska					
Grain yield (bu/acre)	1633	212.84	43.45	41.31	302.80
Nitrogen applied (lbs/acre)	1633	164.99	90.01	0.00	335.00
Nitrogen uptake (grain + stover, lbs/acre)	1483	251.53	62.98	53.00	457.30

Table 3.3. Production function parameter estimates using von Liebig model by state

VARIABLE	STATE					
	Alabama	Georgia	Iowa	Illinois	Mississippi	Nebraska
A. Dry matter yield vs N uptake[a]						
θ_o	25.50**	10.35	-	-	6.410**	64.79***
	(9.20)	(7.92)	-	-	(2.06)	(3.86)
θ_N	0.421***	0.764***	-	-	0.700***	0.603***
	(0.10)	(0.11)	-	-	(0.03)	(0.02)
θ	0.62***	0.86***	-	-	0.75***	0.82***
	(0.05)	(0.05)	-	-	(0.02)	(0.01)
P	78.96***	96.50***	-	-	87.87***	247.0***
	(3.51)	(3.33)	-	-	(0.82)	(1.12)
N^k	127.06***	112.72***	-	-	116.38	302.17***
	(12.50)	(7.83)			(9.02)	(3.60)
No. of obs	80	48	-	-	61	1633
adj. R-sq	0.95	0.98	-	-	0.98	0.98
B. Grain Yield vs N rate applied						
θ_o	45.61***	41.14***	95.12***	74.90***	22.31***	165.5***
	(4.66)	(7.46)	(2.32)	(4.16)	(5.31)	(2.63)
θ_N	0.278***	0.846*	0.401***	0.578***	0.303***	0.404***
	(0.05)	(0.35)	(0.04)	(0.09)	(0.06)	(0.03)
θ	0.63***	1.73***	1.25***	1.24***	0.44***	1.55***
	(0.07)	(0.54)	(0.08)	(0.14)	(0.05)	(0.08)
P	81.71***	80.39***	140.2***	140.3***	72.10***	224.0***
	(5.19)	(2.71)	(1.34)	(2.43)	(6.51)	(1.19)
N^k	129.64***	46.39***	112.37***	113.14***	164.46***	144.77***
	(21.84)	(14.81)	(7.56)	(13.41)	(27.91)	(7.98)
No. of obs	80	48	1998	720	61	1633
adj. R-sq	0.95	0.95	0.90	0.86	0.90	0.97

[a]No available data in Iowa and Illinois

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 3.4. Do kinks line up on a common ray?

STATE	Alabama (θ_{AL})	Georgia (θ_{GA})	Iowa (θ_{IA})	Illinois (θ_{IL})	Mississippi (θ_{MS})	Nebraska (θ_{NE})	All other states
A. Dry matter yield vs *N* uptake							
Alabama (θ_{AL})	-	63.20***	-	-	42.07***	30.99***	99.88***
Georgia (θ_{GA})	63.20***	-	-	-	19.41***	281.63***	99.88***
Iowa (θ_{IA})	-	-	-	-	-	-	-
Illinois (θ_{IL})	-	-	-	-	-	-	-
Mississippi (θ_{MS})	42.07***	19.41***	-	-	-	21.90***	99.88***
Nebraska (θ_{NE})	30.99***	281.63***	-	-	21.90***	-	99.88***
All other states	99.88***	99.88***	-	-	99.88***	99.88***	-
B. Grain yield vs *N* rate applied							
Alabama (θ_{AL})	-	24.81***	40.65***	84.71***	45.20***	41.06***	807.29***
Georgia (θ_{GA})	24.81***	-	0.36	7.02***	98.78***	0.00	807.29***
Iowa (θ_{IA})	40.65***	0.36	-	1.37	9.63***	42.71***	807.29***
Illinois (θ_{IL})	84.71***	7.02***	1.37	-	194.54***	18.72***	807.29***
Mississippi (θ_{MS})	45.20***	98.78***	9.63***	194.54***	-	38.35***	807.29***
Nebraska (θ_{NE})	41.06***	0.00	42.71***	18.72***	38.35***	-	807.29***
All other states	807.29***	807.29***	807.29***	807.29***	807.29***	807.29***	-

The test statistic is distributed as a chi-square.

Table 3.5. Nonnested Hypothesis Results Based on a *P* Test by state

State/	Null Hypothesis			
Alternative Hypothesis	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
ALABAMA				
Linear von Liebig	-	0.14	1.13	0.11
Quadratic	1.48	-	2.45	0.79
Square-root	3.23*	2.47	-	0.36
Mitscherlich-Baule	3.87*	2.26	1.9	-
All alternatives	2.27*	1.54	1.02	1.57
GEORGIA				
Linear von Liebig	-	1.04	0.75	1.5
Quadratic	0.48	-	0.07	0.48
Square-root	0.16	0.07	-	0.11
Mitscherlich-Baule	0.23	0.13	0.01	-
All alternatives	0.1	0.58	0.76	0.82
IOWA				
Linear von Liebig	-	2.7	5.42**	1.28
Quadratic	74.95***	-	4.38**	1.39
Square-root	3.01*	2.62	-	1.83
Mitscherlich-Baule	3.92**	1.8	6.65**	-
All alternatives	1.51	1.81	2.27*	0.69
ILLINOIS				
Linear von Liebig	-	0.4	1.06	5.24**
Quadratic	1.09	-	8.73***	15.32***
Square-root	49.77**	2.59	-	17.55***
Mitscherlich-Baule	18.43***	5.89**	7.86***	-
All alternatives	17.02***	1.55	7.88***	8.09***
MISSISSIPPI				
Linear von Liebig	-	0.00	0.01	0.37
Quadratic	2.07	-	0.06	0.78
Square-root	1.81	0.06	-	0.80
Mitscherlich-Baule	1.93	0.04	0.07	-
All alternatives	0.73	0.03	1.09	0.28
NEBRASKA				
Linear von Liebig	-	2.49	3.15*	0.00
Quadratic	39.45***	-	4.13**	0.01
Square-root	39.20***	4.21**	-	0.01
Mitscherlich-Baule	39.29***	4.40**	2.44	-
All alternatives	6.57***	1.5	0.22	0.04

* p<0.10, ** p<0.05, *** p<0.01

Table 3.6. Production function parameter estimates using von Liebig model, AL

VARIABLE	SITE		
	Belle Mina	Pratville	Thorsby
A. Dry matter yield vs N uptake			
θ_o	2.995	42.22***	15.66**
	(22.50)	(11.34)	(4.57)
θ_N	0.738*	0.124	0.616***
	(0.28)	(0.11)	(0.05)
θ	0.77***	0.28	0.73***
	(0.04)	(0.20)	(0.02)
P	75.78***	76***	97.53***
	(1.26)	(1.62)	(1.96)
N^k	98.63***	272.09**	132.84***
	(6.27)	(132.59)	(4.54)
No. of obs	27	27	26
Adj. R-sq	0.99	0.03	1.00
B. Grain Yield vs N rate applied			
θ_o	46.73***	31.01***	53.64***
	(4.97)	(6.94)	(5.83)
θ_N	0.559***	0.293**	0.310***
	(0.14)	(0.08)	(0.07)
θ	1.48***	0.54***	0.65***
	(0.27)	(0.08)	(0.09)
P	75.09***	67.90***	102.4***
	(1.88)	(7.73)	(6.49)
N^k	50.75***	126.04***	157.24***
	(9.64)	(30.49)	(27.65)
No. of obs	27	27	26
Adj. R-sq	0.99	0.95	0.98

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.7. Production function parameter estimates using von Liebig model, GA

VARIABLE	SITE	
	Tifton	Watskinville
A. Dry matter yield vs N uptake		
θ_o	-11.98	11.5
	(16.91)	(10.01)
θ_N	0.955***	0.793***
	(0.21)	(0.13)
θ	0.85***	0.9
	(0.06)	(0.06)
P	95.18***	98.49***
	(3.56)	(6.21)
N^k	112.21***	109.64***
	(0.43)	(10.14)
No. of obs	18	30
Adj. R-sq	0.99	0.97
B. Grain Yield vs N rate applied		
θ_o	63.61***	38.20***
	(7.45)	(6.41)
θ_N	0.151	0.482***
	(0.09)	(0.08)
θ	0.47**	0.75***
	(0.18)	(0.05)
P	94***	106.8***
	(0.79)	(6.61)
N^k	201.57**	142.19***
	(86.59)	(12.81)
No. of obs	18	30
Adj. R-sq	0.04	0.59

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.8. Production function parameter estimates using von Liebig model, IA

VARIABLE	SITE							
	0	1	10	11	12	13	14	15
Grain Yield vs N rate applied								
θ_o	78.91***	68.19***	118.7***	88.84***	95.01***	88.65***	129.1***	83.60***
	(6.21)	(14.79)	(5.84)	(6.85)	(4.83)	(7.96)	(3.88)	(5.84)
θ_N	0.693***	0.319	0.281***	0.511***	0.328***	0.00956	-0.0591**	0.412***
	(0.13)	(0.17)	(0.06)	(0.09)	(0.06)	(0.12)	(0.02)	(0.09)
θ	1.64***	0.73***	1.06***	1.22***	1.05***	0.14	2.57	1.14***
	(0.19)	(0.26)	(0.17)	(0.14)	(0.14)	(0.50)	(4.63)	(0.17)
P	136.5***	121.4***	161.7***	153.3***	137.9***	95***	126.2***	131.1***
	(2.66)	(13.01)	(4.95)	(4.73)	(3.34)	(2.26)	(4.17)	(3.20)
N^k	83.18***	167.00**	153.18***	126.04***	130.51***	664.52	49.15	115.26***
	(9.95)	(68.88)	(27.72)	(16.41)	(18.63)	(1441.84)	(87.22)	(18.60)
No. of obs	29	61	240	240	240	180	28	28
Adj. R-sq	0.99	0.77	0.93	0.90	0.94	-0.01	1.00	0.99
	16	17	3	4	5	6	8	9
θ_o	150.1***	98.19***	125.0***	22.43***	105.4***	62.92***	65.66***	86.20***
	(6.18)	(5.62)	(6.81)	(5.69)	(4.82)	(5.33)	(6.44)	(3.45)
θ_N	0.407**	0.182**	0.222*	0.349	0.514***	0.694***	0.662***	0.465***
	(0.13)	(0.06)	(0.11)	(0.40)	(0.08)	(0.07)	(0.11)	(0.05)
θ	2.37***	0.68***	1.30***	1.39	1.51***	1.17***	1.26***	1.15***
	(0.51)	(0.17)	(0.45)	(1.09)	(0.16)	(0.08)	(0.13)	(0.07)
P	181.3***	134.1***	150.6***	29.94***	159.8***	154.3***	137.9***	145.0***
	(3.02)	(4.76)	(3.93)	(1.90)	(2.90)	(3.68)	(3.80)	(2.38)
N^k	76.69***	197.32***	115.64***	21.53	105.67***	131.56***	109.07***	126.52***
	(17.30)	(52.23)	(41.33)	(17.21)	(11.97)	(9.80)	(12.35)	(9.11)
No. of obs	60	60	270	30	84	90	88	270
Adj. R-sq	0.99	0.97	0.91	0.02	0.98	0.97	0.96	0.96

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 3.9. Production function parameter estimates using von Liebig model, IL

VARIABLE	SITE	
	Monmouth	ORR
Grain Yield vs N rate applied		
θ_o	106.2***	43.05***
	(4.56)	(5.88)
θ_N	0.587***	0.714***
	(0.11)	(0.11)
θ	1.88***	1.07***
	(0.26)	(0.12)
P	154.4***	128.3***
	(2.63)	(3.42)
N^k	82.03***	119.47***
	(11.87)	(14.66)
No. of obs	330	390
Adj. R-sq	0.937	0.816

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.10. Production function parameter estimates using von Liebig model, MS

VARIABLE	SITE	
	Brooksville	Poplarville
A. Dry matter yield vs N uptake		
θ_o	11.76**	3.425
	(3.32)	(2.99)
θ_N	0.616***	0.752***
	(0.05)	(0.07)
θ	0.71***	0.78***
	(0.03)	(0.04)
P	87.87***	90***
	(4.41)	(2.94)
N^k	123.51***	115.18***
	(8.85)	(10.23)
No. of obs	26	35
Adj. R-sq	0.988	0.835
B. Grain Yield vs N rate applied		
θ_o	30.70***	21.30***
	(5.77)	(5.94)
θ_N	0.249***	0.265***
	(0.05)	(0.05)
θ	0.38***	0.33***
	(0.08)	(0.06)
P	90***	91***
	(0.66)	(0.31)
N^k	238.02***	263.06***
	(25.41)	(34.38)
No. of obs	26	35
Adj. R-sq	0.439	0.365

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.11. Production function parameter estimates using von Liebig model, NE

VARIABLE	SITE							
	Bellwood	Box Butte	Brosius	Brunswick	Cairo	Clay Center	Concord	Funk
A. Dry matter yield vs N uptake								
θ_o	88.91***	71.10*	39.36	103.0***	67.39***	21.53*	79.38*	48.56***
	(6.36)	(27.94)	(25.82)	(14.92)	(12.80)	(10.28)	(33.35)	(13.23)
θ_N	0.608***	0.689***	0.670***	0.429***	0.622***	0.921***	0.465*	0.889***
	(0.03)	(0.17)	(0.14)	(0.09)	(0.05)	(0.07)	(0.20)	(0.07)
θ	0.89***	1.00***	0.84***	0.70***	0.83***	1.01***	0.72***	1.10***
	(0.03)	(0.12)	(0.05)	(0.11)	(0.02)	(0.03)	(0.25)	(0.03)
P	280.1***	226.8***	195.1***	263.5***	267.9***	233.2***	226.1***	253.9***
	(7.66)	(5.70)	(4.38)	(16.35)	(2.61)	(2.73)	(4.17)	(1.97)
N^k	314.67***	225.98***	232.57***	374.40***	322.43***	229.90***	315.79***	231.09***
	(18.47)	(21.88)	(19.16)	(83.13)	(41.09)	(10.21)	(107.09)	(12.49)
No. of obs	96	120	38	144	144	48	144	48
Adj. R-sq	0.999	0.99	0.993	0.99	0.992	0.997	0.992	0.998
B. Grain Yield vs N rate applied								
θ_o	190.0***	216.4***	106.0***	162.9***	168.6***	116.7***	193.8***	169.8***
	(6.49)	(10.12)	(11.64)	(8.55)	(5.93)	(5.13)	(5.60)	(4.39)
θ_N	0.214***	0.0385	0.399***	0.377***	0.425***	0.651***	0.114*	0.716***
	(0.03)	(0.20)	(0.07)	(0.10)	(0.05)	(0.04)	(0.05)	(0.08)
θ	0.77***	0.75	0.90***	1.46***	1.27***	1.28***	1.21***	2.16***
	(0.04)	(2.83)	(0.08)	(0.24)	(0.10)	(0.05)	(0.34)	(0.19)
P	263.0***	228***	189.7***	219.4***	253.7***	236.6***	213.9***	253.9***
	(4.16)	(1.75)	(6.75)	(3.39)	(3.72)	(3.44)	(4.02)	(2.07)
N^k	341.74***	302.50	209.53***	150.00***	200.00***	184.16***	176.89***	117.46***
	(11.94)	(345.48)	(22.89)	(25.23)	(17.12)	(8.41)	(51.79)	(10.64)
No. of obs	96	120	38	144	144	48	144	48
Adj. R-sq	0.317	0.012	0.983	0.976	0.983	0.995	0.984	0.997

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.11. Continued…

VARIABLE	SITE								
	Mead	N. Platte	N. Bend	Paxton	Pickrell	SCAL	Scottsbluff	Spurgin	Wymore
A. Dry matter yield vs N uptake									
θ_o	-9.51	42.77*	134.1*	67.21***	13.82	38.01	73.97***	80.38***	22.95*
	(6.22)	(16.63)	(58.89)	(19.72)	(40.43)	(24.00)	(12.04)	(20.39)	(10.46)
θ_N	0.788***	0.687***	0.367	0.565***	1.080***	0.837***	0.632***	0.523***	0.804***
	(0.03)	(0.09)	(0.33)	(0.11)	(0.28)	(0.12)	(0.05)	(0.09)	(0.07)
θ	0.76***	0.84***	0.92***	0.78***	1.15***	0.98***	0.93***	0.83***	0.92
	(0.01)	(0.04)	(0.20)	(0.08)	(0.06)	(0.04)	(0.03)	(0.06)	(0.03)
P	252.0***	228.6***	222.7***	240.6***	230.8***	254.1***	233.2***	219.2***	189.0***
	(4.21)	(3.12)	(3.67)	(5.37)	(3.30)	(2.94)	(5.16)	(2.65)	(5.69)
N^k	331.65***	270.60***	241.60**	307.14***	201.06***	258.45***	251.92***	265.20***	206.53***
	(8.09)	(12.37)	(94.99)	(34.45)	(20.15)	(13.62)	(10.19)	(7.27)	(7.57)
No. of obs	262	84	36	88	80	144	80	37	40
Adj. R-sq	0.972	0.997	0.996	0.998	0.989	0.996	0.996	0.998	0.997
B. Grain Yield vs N rate applied									
θ_o	159.5***	147.8***	189.5***	191.6***	161.2***	181.7***	189.8***	170.2***	112.3***
	(17.31)	(4.41)	(6.22)	(5.92)	(7.04)	(3.00)	(5.44)	(5.13)	(6.54)
θ_N	0.0965	0.411***	0.267**	0.372***	0.501***	0.746***	0.142***	0.463***	0.570***
	(0.22)	(0.03)	(0.08)	(0.08)	(0.06)	(0.05)	(0.04)	(0.07)	(0.06)
θ	0.59	1.12***	1.78***	1.78***	1.49***	2.54***	1.09***	1.99***	1.42***
	(0.85)	(0.05)	(0.34)	(0.32)	(0.11)	(0.12)	(0.25)	(0.23)	(0.08)
P	191***	233.1***	223.1***	242.1***	242.6***	257.1***	218.1***	221.8***	187.9***
	(1.82)	(3.31)	(3.11)	(2.50)	(5.46)	(1.40)	(5.26)	(1.90)	(3.50)
N^k	326.07	207.32***	125.56***	135.77***	162.46***	101.14***	200.00***	111.51***	132.61***
	(473.51)	(10.61)	(25.08)	(24.61)	(14.27)	(5.13)	(48.91)	(13.09)	(9.00)
No. of obs	262	84	36	88	80	144	80	37	40
Adj. R-sq	0.018	0.994	0.996	0.992	0.988	0.997	0.987	0.998	0.994

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 3.12. Do kinks line up on a common ray for different growing conditions in each state?

HYPOTHESIS	TEST-STATISTIC
A. Dry matter yield vs N uptake	
1. Alabama	18.90***
$\theta_{Be} = \theta_{Pr} = \theta_{Th}$	
2. Georgia	
$\theta_{Ti} = \theta_{Wa}$	2.56
3. Mississippi	
$\theta_{Bv} = \theta_{Po}$	3.20*
4. Nebraska	
$\theta_{Bw} = \theta_{Bb} = \theta_{Bs} = \theta_{Bk} = \theta_{Ca} = \theta_{CC} = \theta_{Co} = \theta_{F} = \theta_{M} = \theta_{NP} = \theta_{NB} = \theta_{Pa} = \theta_{Pk} = \theta_{SC} = \theta_{Sc} = \theta_{Sp} = \theta_{Wy}$	1433.18***
B. Grain yield vs N rate applied	
1. Alabama	
$\theta_{Be} = \theta_{Pr} = \theta_{Th}$	8.97**
2. Georgia	
$\theta_{Ti} = \theta_{Wa}$	0.00
3. Iowa	
$\theta_{Ia0} = \theta_{Ia10} = \theta_{Ia11} = \theta_{Ia12} = \theta_{Ia13} = \theta_{Ia14} = \theta_{Ia15} = \theta_{Ia16} = \theta_{Ia17} = \theta_{Ia3} = \theta_{Ia4} = \theta_{Ia5} = \theta_{Ia6} = \theta_{Ia8} = \theta_{Ia9}$	132.78***
4. Illinois	
$\theta_{Mo} = \theta_{Or}$	0.00
5. Mississippi	
$\theta_{Br} = \theta_{Po}$	0.78
6. Nebraska	
$\theta_{Bw} = \theta_{Bb} = \theta_{Bs} = \theta_{Bk} = \theta_{Ca} = \theta_{CC} = \theta_{Co} = \theta_{Fk} = \theta_{Md} = \theta_{NP} = \theta_{NB} = \theta_{Pa} = \theta_{Pk} = \theta_{SC} = \theta_{Sf} = \theta_{Sp} = \theta_{Wy}$	2506.80***

The test statistic is distributed as a chi-square
*p<0.10, ** p<0.05, *** p<0.01

Table 3.13. Nonnested Hypothesis Results Based on a *P* Test, AL

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Belle Mina				
Linear von Liebig	-	0.33	1.36	0
Quadratic	3.07*	-	2.45	0.03
Square-root	5.96**	2.46	-	0.01
Mitscherlich-Baule	7.14**	2.87	2.95*	-
All alternatives	2.25*	0.95	0.4	0.48
Pratville				
Linear von Liebig	-	0.00	0.23	0.00
Quadratic	0.58	-	0.08	0.04
Square-root	0.04	0.04	-	0.08
Mitscherlich-Baule		0.00	0	-
All alternatives	0.23	0.00	0.15	0.00
Thorsby				
Linear von Liebig	-	0.23	0.29	1.24
Quadratic	7.13**	-	0.32	1.59
Square-root	7.02**	0.32	-	1.38
Mitscherlich-Baule	6.70**	0.3	0.06	-
All alternatives	2.20*	0.1	0.29	0.5

* p<0.10, ** p<0.05, *** p<0.01

Table 3.14. Nonnested Hypothesis Results Based on a *P* Test, GA

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Tifton				
Linear von Liebig	-	4.05*	7.01**	8.78**
Quadratic	1.78	-	2.09	13.56***
Square-root	3.11*	2.02	-	14.93***
Mitscherlich-Baule	3.59*	0.4	0.02	-
All alternatives	4.17**	3.33*	3.84**	5.84**
Watskinville				
Linear von Liebig	-	0.00	0.01	0.07
Quadratic	1.22	-	0.07	0.02
Square-root	1.37	0.07	-	0.00
Mitscherlich-Baule	1.32	0.05	0.10	-
All alternatives	0.43	0.04	0.04	0.05

* p<0.10, ** p<0.05, *** p<0.01

Table 3.15. Nonnested Hypothesis Results Based on a P Test, IA

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
IA0				
Linear von Liebig	-	6.49**	6.97**	13.5***
Quadratic	1.11	-	1.37	4.89**
Square-root	1.43	0.23	-	2.51
Mitscherlich-Baule	3.55*	1.25	1.99	-
All alternatives	1.68	3.55**	3.75**	5.34***
IA1				
Linear von Liebig	-	0.00	0.02	0.03
Quadratic	0.00	-	0	0.03
Square-root	0.47	0.00	-	0.12
Mitscherlich-Baule	0.06	0.01	0	-
All alternatives	0.45	0.03	0.08	0.05
IA10				
Linear von Liebig	-	0.24	0.34	0.01
Quadratic	1.57	-	0.11	0.18
Square-root	0.8	0.12	-	0.12
Mitscherlich-Baule	1.38	0.2	0.02	-
All alternatives	0.6	0.13	0.32	0.14
IA11				
Linear von Liebig	-	0.33	0	0.59
Quadratic	0.46	-	0.23	0.49
Square-root	0.52	0.27	-	0.58
Mitscherlich-Baule	0.71	0.57	0	-
All alternatives	0.28	0.26	0.58	0.24
IA12				
Linear von Liebig	-	0.02	0.22	0.06
Quadratic	1.51	-	0.19	0.38
Square-root	0.68	0.1	-	0.66
Mitscherlich-Baule	1.34	0.37	0	-
All alternatives	0.81	0.37	0.67	0.39

* p<0.10, ** p<0.05, *** p<0.01

Table 3.15. Continued….

Alternative Hypothesis	Null Hypothesis: Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
IA13				
Linear von Liebig	-	0.00	0.01	1.13
Quadratic	2.66	-	0.12	0.94
Square-root	2.47	0.11	-	1.02
Mitscherlich-Baule	2.4	0.07	0.00	-
All alternatives	1.01	0.13	0.1	0.41
IA14				
Linear von Liebig	-	0.12	0.11	0.31
Quadratic	0.00	-	0.11	0.19
Square-root	0.1	0.02	-	0.21
Mitscherlich-Baule	0.00	0.00	0.32	-
All alternatives	1.23	1.27	3.64**	1.34
IA15				
Linear von Liebig	-	0.32	0.62	0.27
Quadratic	0.52	-	1.13	0.01
Square-root	4.39**	4.07*	-	0.35
Mitscherlich-Baule	4.54**	3.68*	0.46	-
All alternatives	1.53	1.29	0.09	0.15
IA16				
Linear von Liebig	-	3.10*	1.37	0.5
Quadratic	0.11	-	3.89*	1.24
Square-root	0.23	1.77	-	0.04
Mitscherlich-Baule	0.12	2.6	0.49	-
All alternatives	0.15	1.12	1.01	0.49
IA17				
Linear von Liebig	-	0.00	0.00	0.03
Quadratic	0.52	-	0.14	0.02
Square-root	0.76	0.3	-	0.12
Mitscherlich-Baule	0.77	0.18	0.12	-
All alternatives	0.39	0.25	0.22	0.2

* p<0.10, ** p<0.05, *** p<0.01

Table 3.15. Continued….

Alternative Hypothesis	Null Hypothesis: Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
IA3				
Linear von Liebig	-	0.39	0.04	0.15
Quadratic	0.00	-	0.08	0.00
Square-root	0.10	0.19	-	0.01
Mitscherlich-Baule	0.05	0.28	0.00	-
All alternatives	0.07	0.20	0.16	0.11
IA4				
Linear von Liebig	-	1.24	0.02	0.08
Quadratic	0.15	-	0.08	0.15
Square-root	0.00	0.98	-	0.00
Mitscherlich-Baule		1.24	0.02	-
All alternatives	0.09	0.60	0.41	0.09
IA5				
Linear von Liebig	-	0.58	0.42	0.35
Quadratic	2.61	-	1.76	0.05
Square-root	6.35**	5.69**	-	0.4
Mitscherlich-Baule	6.42**	5.16**	0.10	-
All alternatives	2.23*	1.9	0.18	0.24
IA6				
Linear von Liebig	-	3.03*	0.38	4.57**
Quadratic	0.05	-	0.07	1.7
Square-root	0.07	0.02	-	4.49**
Mitscherlich-Baule	0.22	0.33	0.25	-
All alternatives	0.28	1.21	3.02**	1.72
IA8				
Linear von Liebig	-	2.24	0.08	0.74
Quadratic	0.13	-	0.9	0.23
Square-root	1.00	1.69	-	0.32
Mitscherlich-Baule	0.92	2.52	0.03	-
All alternatives	0.33	1.02	0.73	0.27
IA9				
Linear von Liebig	-	0.21	0.28	0.50
Quadratic	2.58	-	1.31	1.06
Square-root	5.45**	3.25*	-	0.20
Mitscherlich-Baule	6.11**	3.71*	0.03	-
All alternatives	2.10	1.23	1.33	0.37

* p<0.10, ** p<0.05, *** p<0.01

Table 3.16. Nonnested Hypothesis Results Based on a *P* Test, IL

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Monmouth				
Linear von Liebig	-	2.51	0.26	0.06
Quadratic	0.00	-	1.31	0.01
Square-root	0.15	2.46	-	0.06
Mitscherlich-Baule	0.14	2.57	0.38	-
All alternatives	0.08	0.87	0.09	0.04
ORR				
Linear von Liebig	-	1.27	0.82	0.00
Quadratic	2.46	-	0.30	0.08
Square-root	1.84	0.75	-	0.00
Mitscherlich-Baule	0.07	0.56	0.26	-
All alternatives	2.64**	1.21	4.08***	0.00

* p<0.10, ** p<0.05, *** p<0.01

Table 3.17. Nonnested Hypothesis Results Based on a *P* Test, MS

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Brooksville				
Linear von Liebig	-	0.58	1.08	0.26
Quadratic	7.85**	-	1.78	0.55
Square-root	8.30***	1.32	-	0.21
Mitscherlich-Baule	8.48***	1.8	0.65	-
All alternatives	3.42**	0.97	0.4	0.55
Poplarville				
Linear von Liebig	-	0.56	0.38	0.59
Quadratic	0.04	-	0.01	0.11
Square-root	0.11	0.01	-	0.02
Mitscherlich-Baule	0.05	0.06	0.01	-
All alternatives	0.68	0.28	0.36	0.44

* p<0.10, ** p<0.05, *** p<0.01

Table 3.18. Nonnested Hypothesis Results Based on a *P* Test, NE

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Bellwood				
Linear von Liebig	-	1.59	1.52	2.15
Quadratic	3.06*	-	0.62	1.29
Square-root	3.27*	0.62	-	1.06
Mitscherlich-Baule	3.27*	0.49	0.61	-
All alternatives	1.34	0.54	0.82	1.03
Box Butte				
Linear von Liebig	-	0.27	0.43	0.17
Quadratic	4.15**	-	1.01	0.22
Square-root	4.33**	1.01	-	0.23
Mitscherlich-Baule	4.39**	0.99	0.94	-
All alternatives	1.77	0.35	0.14	0.08
Brosius				
Linear von Liebig	-	0.17	0.13	0.16
Quadratic	9.70***	-	0.01	0.62
Square-root	8.15***	0.01	-	0.57
Mitscherlich-Baule	8.35***	0.02	0.01	-
All alternatives	3.21**	0.11	0.42	0.37
Brunswick				
Linear von Liebig	-	0.00	0.35	1.48
Quadratic	5.02**	-	4.39**	5.64**
Square-root	7.51***	4.40**	-	5.43**
Mitscherlich-Baule	6.52**	1.77	4.87**	-
All alternatives	4.61***	3.18**	2.29*	2.84**
Cairo				
Linear von Liebig	-	2.47	2.14	5.54**
Quadratic	1.80	-	1.03	4.20**
Square-root	1.98	1.03	-	1.9
Mitscherlich-Baule	2.02	0.80	0.46	-
All alternatives	0.71	0.88	2.09	1.84

* p<0.10, ** p<0.05, *** p<0.01

Table 3.18. Continued….

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Clay Center				
Linear von Liebig	-	1.51	2.12	4.55**
Quadratic	3.77**	-	3.36*	8.01***
Square-root	2.73	3.31*	-	7.57***
Mitscherlich-Baule	3.06*	1.82	3.08*	-
All alternatives	2.36*	1.61	3.51**	2.95**
Concord				
Linear von Liebig	-	5.72**	4.94**	9.04***
Quadratic	0.04	-	0.49	8.00***
Square-root	0.00	0.45	-	7.35***
Mitscherlich-Baule	0.00	0.17	1.46	-
All alternatives	1.3	2.93**	3.15**	3.01**
Funk				
Linear von Liebig	-	0.31	0.3	2.38
Quadratic	0.86	-	0.2	2.45
Square-root	0.82	0.20	-	2.62
Mitscherlich-Baule	0.73	0.15	0.07	-
All alternatives	0.28	0.1	0.37	2.04
Mead				
Linear von Liebig	-	3.55*	2.98*	0.00
Quadratic	0.79	-	0.68	0.04
Square-root	1.46	0.68	-	0.00
Mitscherlich-Baule	0.00	0.00	0.04	0.15
All alternatives	0.00	0.15	0.00	0.04
N. Platte				
Linear von Liebig	-	0.29	0.18	2.11
Quadratic	8.37***	-	0.00	1.34
Square-root	8.89***	0.00	-	0.79
Mitscherlich-Baule	9.00***	0.00	0.01	-
All alternatives	3.05**	0.31	0.71	0.75

* p<0.10, ** p<0.05, *** p<0.01

Table 3.18. Continued….

Alternative Hypothesis	Null Hypothesis: Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
North Bend				
Linear von Liebig	-	0.50	0.70	0.22
Quadratic	3.14*	-	1.40	0.60
Square-root	3.69*	1.40	-	0.57
Mitscherlich-Baule	4.08*	0.00	1.29	-
All alternatives	1.65	0.65	0.49	0.32
Paxton				
Linear von Liebig	-	1.15	1.10	0.37
Quadratic	52.01***	-	0.82	0.01
Square-root	49.86***	0.82	-	0.01
Mitscherlich-Baule	50.36***	0.92	0.38	-
All alternatives	17.03***	0.73	0.41	0.69
Pickrell				
Linear von Liebig	-	1.20	1.19	3.31*
Quadratic	0.94	-	0.16	2.53
Square-root	0.46	0.16	-	2.96*
Mitscherlich-Baule	0.11	0.06	0.01	-
All alternatives	0.48	0.41	0.41	1.97
SCAL				
Linear von Liebig	-	1.13	0.78	3.45*
Quadratic	29.44***	-	0.47	1.14
Square-root	27.04***	0.47	-	1.18
Mitscherlich-Baule	25.99***	0.43	0.3	-
All alternatives	11.48***	0.94	0.82	1.24
Scottsbluff				
Linear von Liebig	-	0.05	0.00	0.57
Quadratic	2.67	-	1.86	1.73
Square-root	3.54*	1.84	-	1.91
Mitscherlich-Baule	3.18*	0.98	2.31	-
All alternatives	2.95**	1.82	1.52	1.81

* p<0.10, ** p<0.05, *** p<0.01

Table 3.18. Continued….

Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
Spurgin				
Linear von Liebig	-	0.01	0.00	0.09
Quadratic	2.72	-	0.41	1.32
Square-root	2.13	0.41	-	1.26
Mitscherlich-Baule	1.87	0.29	0.07	-
All alternatives	1.67	0.72	0.92	1
Wymore				
Linear von Liebig	-	0.02	0.06	0.04
Quadratic	8.51***	-	0.44	0.07
Square-root	9.04***	0.44	-	1.3
Mitscherlich-Baule	9.17***	0.57	1.12	-
All alternatives	5.19***	0.92	0.71	1.83

* p<0.10, ** p<0.05, *** p<0.01

Figure 3.1. Relation of total dry matter yield to total N uptake
for corn experiments

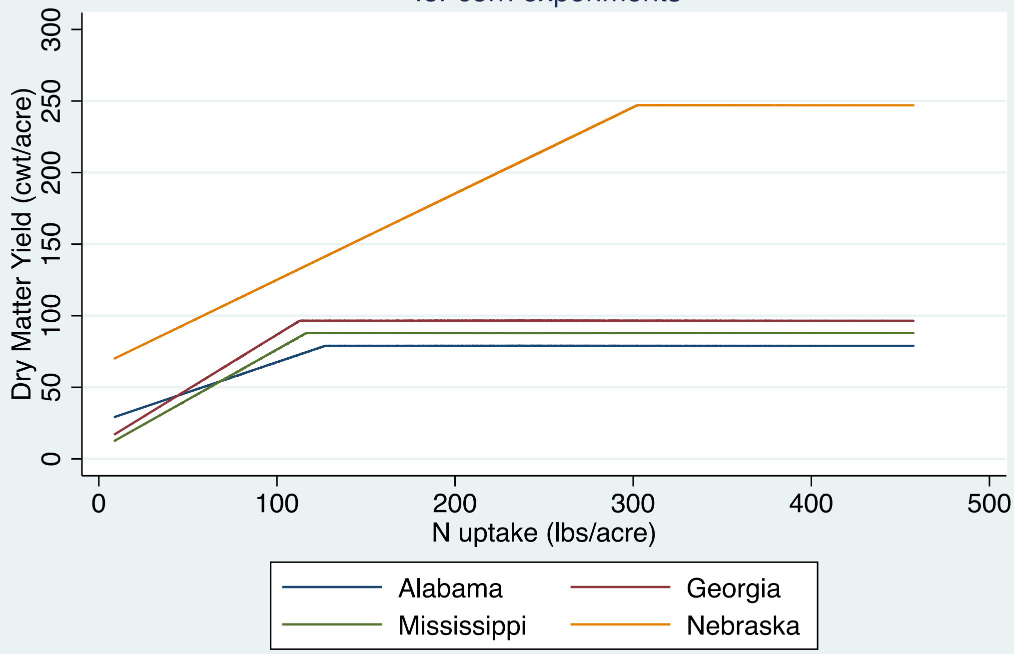
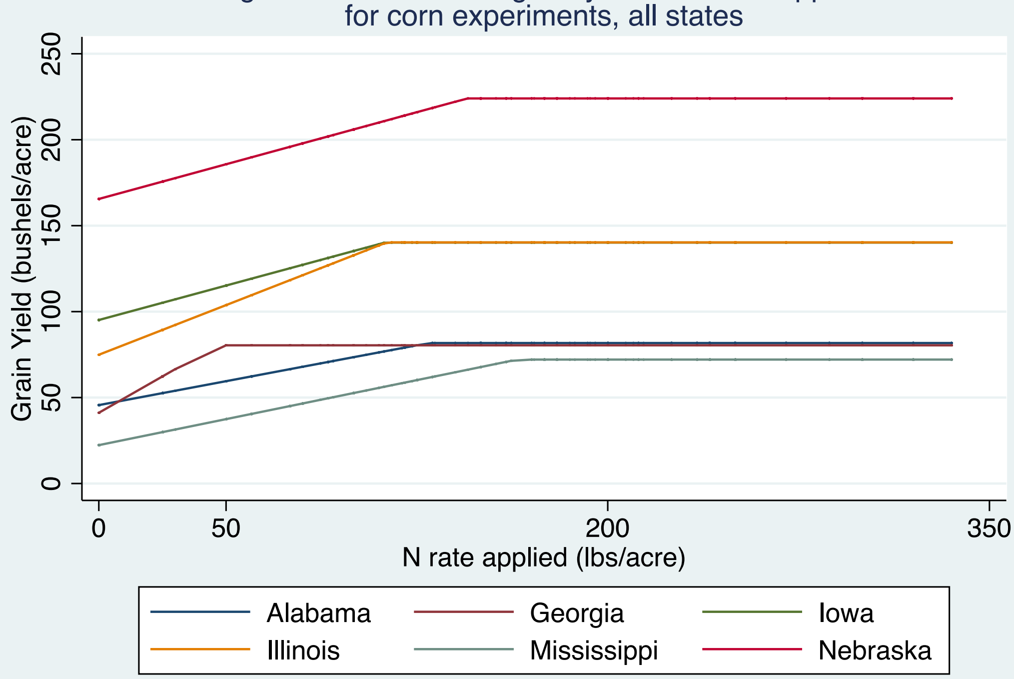


Figure 3.2. Relation of grain yield to total N applied
for corn experiments, all states



Chapter 4

AN ASSESSMENT OF THE SITE-SPECIFIC NUTRIENT MANAGEMENT (SSNM) FOR IRRIGATED RICE IN ASIA

Rice is an important staple food for about 70 percent of the Asian population (nearly 3 billion people). More than 75 percent of rice worldwide is produced in irrigated rice lands, and 90 percent of these irrigated lands are paddy rice production found predominantly in Asia (Bouman *et al.*, 2006). Irrigated rice grown under favorable tropical conditions requires essential nutrients such as *N*, *P*, and *K* that are typically not present in the soil in sufficient amounts to meet crop needs. Nitrogen is the most important nutrient because it significantly affects tillering, leaf area growth, biomass production, and grain yield (Yang, 2003). Crop scientists also consider it the most "limiting" agent in almost all soils (Balasubramanian *et al.,* 1999).

Fertilizer recommendations provided to farmers typically do not consider field, climate, and management-specific effects on the nutrient needs of the crop (Buresh, 2006). Indeed, factors affecting crop yield and quality are site-specific (Reets and Fixen, 2000). Therefore, to ensure that *N* and other essential plant nutrients are provided in optimal amounts and are readily available during crop growth periods, site-specific nutrient management (SSNM) was developed in Asia by the International Rice Research Institute (IRRI). Unlike other fertilizer recommendation algorithms that are often derived from factorial fertilizer trials conducted across multiple locations, SSNM is an "alternative approach for dynamic management of nutrients to optimize supply and demand of a nutrient within a specific field in a particular cropping season" (Dobermann et al., 2004). SSNM defines the optimal amounts of nitrogen and other essential plant nutrients as the amounts that maximize yield. The underlying premise of SSNM is that if nutrients are applied to crops at appropriate times and rates, then the use of indigenous and applied nutrients will be optimized.[23] As a result, wider farmer adoption of SSNM will increase land productivity, yield, and profitability of farmers, and decrease fertilizer-related pollution in the environment (Buresh, 2006). SSNM strategy, through the use of *Nutrient Manager for Rice*, a computer- and mobile phone-based application that provides farmers with fertilizer advice matching their particular farming conditions, is being practiced in Bangladesh, Guangdong, China, Tamil Nadu, India, Indonesia, Philippines, and West Africa (IRRI, 2012). Further work is being conducted to make this decision-tool available on mobile devices in other countries (e.g. Vietnam).

Like Stanford's 1.2 Rule, the SSNM fertilizer recommendations are based on the yield goal approach. From the previous chapter, I showed that yield goal base approach only makes economic sense if the crop production satisfies two restrictions: (1) it is of the von Liebig functional form, and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with a constant slope. Although there are studies that assessed the impacts of SSNM strategy in rice (e.g. Dobermann et al. 2002, Pampolino et al., 2007; Flor 2011; Rodriguez and Nga 2012), there are no studies that critically discuss and investigate some of the assumptions underlying the SSNM and its current NPK fertilizer recommendation algorithm, and assess its scope for improving irrigated rice management. Therefore, the objective of this paper is to discuss and evaluate the principles of SSNM research. I emphasized an underlying assumption about the relationship among major nutrients and soil organic matter, and the assumption's implications for fertilizer recommendation. I explored whether major nutrients are technically substitutes or complements, and whether ex ante soil conditions matter to the return on investments in inorganic fertilizer, in particular *N* fertilizer.

[23] SSNM strategy offers proper timing and splitting patterns of fertilizer applications through the use of a location-specific nutrient splitting scheme or tools such as leaf color chart.

4.1. The SSNM Strategy for Rice

Based on the yield goal approach for fertilizer recommendation, the SSNM strategy for rice requires information on farmer's yield goal, indigenous supply of *N*, *P*, and *K* and the crop nutrient requirements. Season-specific yield goals are set in the range of 70-80 percent of potential yield.[24] Crop nutrient requirements for a specific yield goal are then quantified using the empirical modeling approach in Quantitative Evaluation of the Fertility Tropical Soils (QUEFTS) (Jansen et al., 1990). The QUEFTS principles can be expressed simply in an equation as

$$F_X = \frac{U_X - U_{X_{0X}}}{E_{F_X}} \tag{4-1}$$

where X is one of the three macronutrients *N*, *P*, or *K*, F_X (kg per ha) is the fertilizer nutrient requirement to achieve a specified yield target, U_X is the predicted optimal nutrient uptake requirement for the specified yield target (kg per ha), $U_{X_{0X}}$ is the indigenous nutrient supply, and E_{Fx} is the agronomic efficiency of fertilizer *X*. The indigenous nutrient supplies of *N*, *P*, and *K* are each defined as the total amount of that nutrient available to the crop from the soil during a cropping cycle, when other nutrients are non-limiting. It is estimated by measuring plant nutrient uptake in an omission plot. For example, the indigenous *N* supply can be measured as plant *N* uptake at harvest in a small 0-*N* plot located in a farm field, where *P*, *K*, and other nutrients are supplied in sufficient amounts so that plant growth is limited only by the indigenous *N* supply. This is one distinct characteristic of the SSNM approach, i.e. use of crop-based estimates of the indigenous nutrient supply instead of relying on soil tests. Hence equation (4-1) can be expressed using yield gain-based approach algorithm:

[24] Potential yield can be defined as grain yield limited by climate and genotype only, with all other factors not limiting crop growth.

$$F_X = \frac{\left(Y_G - Y_{GX_{0X}}\right)U'_X}{E_{F_X}} \qquad (4\text{-}2)$$

where Y_G reflects the total amount of *N, P,* or *K* nutrient that must be taken up by the crop to achieve the yield goal or target yield (Y_G), $Y_{GX_{0X}}$ is *X nutrient*-limited yield or grain yields attainable from the indigenous supply of *X* nutrient, U'_X (a constant[25]) is the optimal plant nutrient uptake requirement of *N*, *P*, or to produce a ton of grain yield, and E_{Fx} is the agronomic efficiency of fertilizer *X*. Location-specific fertilizer requirements can be calculated for most irrigated rice areas based on the expected yield increase over the respective omission plot and using certain assumptions on plant nutrient requirements and fertilizer efficiency of applied fertilizer nutrients. The QUEFTS model predicts a linear increase in grain yield if nutrients are taken up in balanced amounts of 14.7 kg *N*, 2.6 kg *P*, and 14.5 kg *K* (*UX'*, equation 4-2) per one ton of grain yield produced, until the yield reaches about 70-80 percent of the potential yield (Witt et al., 1999). Similar to the Stanford's 1.2 Rule, this algorithm is simple with minimal characterization or interviewing of farmers for each field, in order to ensure rapid, cost-effective delivery of field-specific guidelines to millions of small-scale farmers (Buresh et al., 2010).

By estimating a quadratic production function, I investigated two research questions:

(1) Is there evidence of complementary, von Liebig type relationships among *N* fertilizer, *P* fertilizer, and *K* fertilizer?

(2) Does yield response to *N* fertilizer application depend on the initial state of the soil?

A focus on agronomically optimal nutrient application rates can be misleading if it fails to note the importance of interaction between inputs, whether inputs are substitutes, complements, or

[25] The nutrient requirement is only a constant if yield goals are chosen that are equal to or lower than 70 to 80 percent of the potential yield.

independent. Understanding nutrient interactions may provide explanation as to why farmers over- or under-apply nutrients. While IRRI scientists acknowledge that deficiency of any one nutrient will impair the crop uptake and utilization of the other nutrients, there are no studies on SSNM that have confirmed if there are indeed von Liebig type complementarities among the major soil nutrients: *N, P* and *K.* If nutrients exhibit a von Liebig type relationship, a given level of yield can only be attained by use of single combination of inputs. In this case allocations under profit maximization, which account for relative input and output prices, can be ignored in the fertilizer algorithm. Equation (4-2) suggests that input and output prices will not affect the amount of fertilizer recommendation. SSNM algorithm only makes economic sense if indeed the rice crop production function is linear von Liebig. The economically optimal fertilizer application rate is the minimum rate at which rice yield reaches its plateau. Any change in the ratios of input prices does not affect the fixed proportion in which inputs are optimally combined in the production process.

Input and output prices, however, affect production decisions of farmers. When farmers are faced with cash constraints and if there are differences in availability and price of single fertilizer due to differential subsidy levels, they tend to buy and use mostly *N* fertilizers (Balasubramanian, 1999). Dawe (1998) also reports that the declining yield growth rates in double- and triple-crop rice monocropping systems were partly due to lower rice prices. In this situation, input and output prices cannot be ignored in making fertilizer recommendations. In addition, if *N* fertilizer is applied alone, *P* becomes a limiting element after a few years of intensive cultivation with high doses of *N* and *P* application (Balasubramanian, 1999). If *P* becomes limiting in the soil and if indeed *N* and *P* are complements, adding more *N* fertilizer will not be beneficial for crops.

Note also that the existing SSNM algorithm does not take into account the possible relationship of *N* fertilizer application and soil organic matter, as reflected in soil carbon, *C*, contents. The rationale behind this is that previous studies show that indigenous *N* supply was quite variable among fields and not related to soil organic matter content (Cassman, et al. 1996). Organic fertilizers in smallholder agriculture do not add nutrients to a cropping system as a whole, but rather are a means of nutrient transfer (Dobermann, 2004). The organic amendments when used as a complement to inorganic *NPK* increase yields but that increased yields are due to increased nutrient supply (*N*, *P*, *K*, or other nutrients under conditions of deficient soil nutrient supply) and not the "organic matter effect" (Dawe et al 2003).

A few studies show, however, that increasing soil organic matter in terms of soil *C* content makes fertilizer application of *N* more effective and can improve crop yields (Tiessen et al., 1994; Marenya and Barrett, 2009). Soil organic matter contributes to soil quality and ecosystem function through its influence on soil physical stability, soil microbial activity, nutrient storage and release, and environmental quality (Herrick and Wander, 1997). Increasing organic *C* content in soil organic matter reduces soil erosion and degradation, improves surface water quality, and increases soil productivity. Some studies also suggest that soil organic matter content increases under inorganic fertilization, especially for inorganic *N* fertilizers (Majumder et al., 2008 and Reid, 2008). Lopez-Bellido et al. (2010) and Luo et al. (2010) suggest that soil organic content does not change, and others suggest that it in fact decreases under inorganic fertilization (Manna et al., 2006, Khan et al., 2006; Li and Zhang, 2007).

I hypothesized that the marginal physical product (MPP), and thus the profitability, of *N* fertilizer application depends on soil *C* stocks, which may vary systematically in farmers' fields.

As an initial test, I used kernel-weighted local polynomial regression[26] to check if rice yields are strongly and directly associated with soil *C* stocks (Figures 4.1-4.8). With the exception of Suphan Buri, Thailand, there is clear evidence that grain yield increases as soil organic matter, as represented by organic *C* content, increases. The marginal returns to fertilizer application may vary with soil organic matter. For these reasons, there is a great need to quantify the role of soil organic matter, particularly the soil *C* stocks, in relation to crop output response to *N* fertilizer in irrigated rice systems. The complementary between soil organic matter and *N* fertilizer application might mean that *N* fertilizer application becomes unprofitable on soils depleted of soil organic matter (Marenya and Barrett, 2009). Poor soil fertility might actually be a cause, not merely a consequence, of low rates of fertilizer use (Morris et al., 2007). If this is the case, then *ex ante* soil conditions matters a lot to the return on investments in fertilizer policies (Marenya and Barrett, 2009). In cases where soil degradation has become severe, provision of temporary fertilizer subsidies or cost-shares might not be an appropriate policy.

4.2. Literature Review

Most of the published studies on SSNM specifically examined its impact on fertilizer and/or paddy yields, primarily using field experiments. Dobermann *et al*. (2002) conducted on-farm experiments from 1997 to 1999 to develop and test a new SSNM approach for eight key irrigated rice production domains of Asia located in six countries. They found that average grain yield increased by 0.36 Mg per hectare with SSNM as compared to current farmers' fertilizer practice in their study in cropping systems in Asia. Their results also showed that SSNM led to significant increases in nitrogen use efficiency. In terms of profitability, on average, across all

[26] Weighted least squares regression is used to fit linear or quadratic functions of the predictors at the centers of neighborhoods (Cleveland, 1979). One chief attraction of this regression is that I do not need to specify a function of any form to fit a model to the data, only to fit segments of the data.

sites, there was an increase in profitability of US$46 per hectare through the use of SSNM. Son *et al*. (2004) particularly analyzed the SSNM in irrigated rice systems of the Red River Delta. A SSNM plot was established on each of the 24 farm fields as a comparison with the farmers' fertilizer practice. SSNM increased yield by 0.19 tons per hectare over the farmers' field practice, decreased the total fertilizer cost by about $2 per hectare in 1998 and by $22 per hectare in 1999, and increased average farm profits by $41 per hectare in 1998 and $74 per hectare in 1999.

Pampolino *et al*. (2007) explored not only the economic benefits of SSNM but also its environmental impacts. SSNM led to higher efficiency of nitrogen use. SSNM decreased the percentage of total nitrogen losses from applied fertilizers, thus reducing the nitrous oxide emissions and global warming. Economic performance of SSNM adopters and non-adopters were also compared using economic data through focus group discussions. Gross revenue and gross return above fertilizer costs were higher for SSNM than non-SSNM farmers across the three countries. Dawe et al. (2004) found in their study in China, Southern India, and the Philippines the profitability in SSNM ranged from $57 to $82 per hectare. They also found out that the sites in Vietnam (southern and northern) exhibited intermediate levels of profitability at $38-39 per hectare. Studies of Khurana et al. (2007) in northwestern India and Wang et al. (2007) in China found similar results. In 2009, Buresh et al. provided alternatives to factorial field trials and rigid nutrient balances for determining fertilizer *K* and *P* requirements in the SSNM strategy. However, their proposed framework did not specifically consider soil-plant-nutrient interactions and biological processes mediating nutrient availability.

Published reports on SSNM tend to be optimistic. There are no reports that critique the SSNM approach to fertilizer recommendation, and very few assess its scope for improving irrigated rice production. This paper contributes to the literature by providing a broader scope of analysis of SSNM in the irrigated rice systems by exploring whether there are indeed interactions among essential nutrients *N*, *P*, and *K* and whether complementarities between soil organic matter and applied *N* might mean that fertilizer application becomes unprofitable on soils with low soil organic matter.

4.3. Data and Empirical Model

4.3.1. Data

The data on irrigated rice production and input use come from the IRRI project on *Reversing Trends of Declining Productivity in Intensive Irrigated Rice Systems* (RTDP) in six countries across tropical and subtropical environments in Asia (Table 4.1a). In each of the six countries, data originated from both nutrient omission trials and fertilizer evaluation trials conducted in farmers' fields (Dobermann et al., 2002). The treatments that were used in the study are: (1) no fertilizer applied (0 *N*, 0 *P*, 0 *K*), (2) *PK* applied, 0 *N* applied, (3) SSNM, and (4) farmer's field practice with no interference by IRRI. All data were for irrigated rice, and water rarely limited plant growth. The 0-*N* plots received 30 kg *P* fertilizer and 50 kg *K* fertilizer per hectare. The 0-*N*, 0-*P*, and 0-*K* treatments were separated from the surrounding field by bunds and were moved to a different location after each crop grown, to avoid residual effects caused by nutrient depletion. Each experiment in six countries was run for three to five years. I only used data for one year in some areas and two years in some areas because of data availability. Each treatment contained two to three replicate sampling plots per farm. The semi-dwarf, modern high-yielding *indica* cultivars were grown with good agronomic practices. Comparable methodologies for plant sampling, yield determination, and analysis for plant nutrients were used for collected data across countries and experiments (Witt et al., 1999). Soil data were collected at the single field/single treatment level, that is, only for the field used for the agronomic research. Two 6x6 m plots were sampled for each treatment and the samples were processed separately. The total organic *C* of soil samples from 0-*N* plots was determined based on Walkley (1947).

The sample farmers at each site were selected based on the following criteria: (1) represent the most common soil types in the region, (2) represent the most typical cropping systems and farm management practices in the region, (3) represent a range of socioeconomic conditions (small to large farms, poor to rich farmers), (4) reasonable accessibility to allow frequent field visits, and (5) farmer interest in participating in the project over a longer term. Socio-economic data were collected at the whole-farm level, i.e. including the field used for the agronomic research as well as other fields belonging to the same farmer.

4.3.2. Model

The rice production function for each experimental site can be defined by using a generalized quadratic specification (Chambers, 1988)[27]:

$$y = \beta_0 + \sum_{i=1}^{m} \beta_i x_i + \sum_{i=1}^{m} \sum_{j=1}^{m} \beta_{ij} x_i x_j + e \qquad (4\text{-}3).$$

Here y is grain yield, x_i is the vector of independent variables – *N* applied, *P* applied, *K* applied, and soil *C* stocks, age of farmers, farm area harvested, and dummy variable for high yielding season (HYS) – the β vector comprises the parameter estimates of interest and e is an iid N(0,

[27] Berck and Helfand (1990) show that in the presence of heterogeneity, the polynomial and linear plus plateau approximations essentially converge, making the quadratic a viable alternative to the von Liebig and linear response plateau models. Moreover, Berck et al. (2000) find that von Liebig models generally do not fit the data well and that actual estimation does not yield the right angle isoquants described in its derivation.

σ^2) error term.[28] In order to explore the systematic relationship among fertilizer *NPK* application and ex-ante soil fertility in each experimental site, I tested the null hypothesis that *N* fertilizer, *P* fertilizer, and *K* fertilizer do not significantly interact with each other and that soil *C* content has no indirect effects on yields through *N* fertilizer application:

$$\begin{aligned} H_0 &: \beta_{ij} = 0 \\ H_a &: \beta_{ij} \neq 0 \end{aligned} \qquad (4\text{-}4).$$

A Wald test was performed to test the joint significance of parameters β_{ij} in equation (4-3) for each study site. If H_0 cannot be rejected, $\frac{\partial^2 y}{\partial x_i \partial x_j} \equiv \frac{\partial}{\partial x_i}\left(\frac{\partial y}{\partial x_j}\right) \equiv \beta_{ij} \equiv \beta_{ji} \equiv 0$, then it indicates independence of x_i and x_j. That is, the marginal productivity of x_j is not affected by changes in the level of x_i. If, however, H_0 is rejected, then nutrient interaction between x_i and x_j is present. If $\beta_{ij} \equiv \beta_{ji} > 0$, then x_i and x_j are technically complementary. The marginal product of x_i increases as x_j increases. If $\beta_{ij} \equiv \beta_{ji} < 0$, then x_i and x_j are technically substitutes. Increasing x_i reduces the marginal productivity of x_j. Tables 4.1b and 4.1c show the definition and summary of statistics, respectively, for the regression variables.

As in Chapter 2, I used the non-nested hypothesis framework proposed by Davidson and MacKinnon (1983) to contrast the quadratic model (equation 4-3) against the linear von Liebig model,

$$y = \min\{\theta_0 + \theta_N N, \theta_1 + \theta_P P, \theta_3 + \theta_K K, M\} + e \qquad (4\text{-}5),$$

non-linear von Liebig model,

[28] The *N*, *P*, and *K* fertilizer application of farmers could be endogenous given by the unobserved factors that affect yields. There are no good instruments available to address endogeneity concerns in the production function estimation.

$$y = \min\{\theta_0 + \theta_N N + \theta_{NN} N^2, \theta_1 + \theta_P P + \theta_{PP} P^2, \theta_3 + \theta_K K + \theta_{KK} K^2, M\} + e \qquad (4\text{-}6),$$

and square-root model,

$$y = \beta_0 + \sum_{i=1}^{m} \beta_i x_i + \sum_{i=1}^{m} \sum_{j=1}^{m} \beta_{ij} \sqrt{x_i x_j} + e \qquad (4\text{-}7)$$

in all the study sites in the study.[29]

The P test may also be applied to the case of multivariate nonlinear regression models. The null hypothesis may be written as:

$$H_0 : y_{it} = m_{it}(\theta) + u_{it}^0, \qquad u_t^0 \sim N(0, \Omega_0), \qquad (4\text{-}8)$$

and the alternative as:

$$H_1 : y_{it} = g_{it}(\beta) + u_{it}^1, \qquad u_t^1 \sim N(0, \Omega_1). \qquad (4\text{-}9)$$

Here i indexes the n equations, t indexes the T observations, and Ω_j , j=1,2, is the nxn contemporaneous covariance matrix for the error terms corresponding to hypothesis H_j. Analogous to equation (2-15) in the univariate case in Chapter 2, the artificial compound model is

$$H_C : y_{it}(1-\alpha) m_{it}(\theta) + \alpha \hat{g}_{it} + u_{it} \qquad (4\text{-}10)$$

where, under H_0, the vector u_t should have covariance matrix Ω_0. Linearizing (4-10) around the point $\alpha = 0$, $\theta = \hat{\theta}$ yields the multivariate linear regression

$$y_{it} = \hat{m}_{it} = \hat{M}_{it}^T \theta + (\hat{g}_{it} - \hat{m}_{it}) + u_{it} \qquad (4\text{-}11)$$

where m_{it} and g_{it} denote $m_{it}(\hat{\theta})$ and $g_{it}(\hat{\beta})$ and $\hat{M}_{it}$ denotes the vector of derivatives of $m_{ti}(\theta)$ with respect to θ, evaluated at $\hat{\theta}$. This regression is to be estimated by generalized least

[29] M is the plateau yield.

squares, using $\hat{\Omega}$ as the assumed covariance matrix, and is referred to as the P_0 test. The test statistic is the t statistic on $\hat{\alpha}$.

4.4. Results and Discussion

I estimated two variations of my basic model. In Model 1, I only used the nutrients as controls in the production function. In Model 2, I included a high-yielding season dummy (HYS) and farm area. I favored Model 2 over Model 1 for all study sites and only discussed those results. The addition of controls, HYS and farm area, in Model 2 proved to be statistically significant when included in the regression.[30]

Tables 4.2-4.4 report the OLS regression results from equation (4-3) by study site. Across all sites, not all the significant coefficient estimates have the expected signs in the first-order term. The expected rice yield is decreasing in soil organic *C* content in Can Tho, Vietnam. It is possible that large amounts of organic materials repeatedly applied on soil with lower buffering capacity and high reducible iron content may cause acceleration in soil reduction and thereby potential iron toxicity in rice (Ponnamperuma, 1972). Literature suggests that plants suffering from iron toxicity may cover large contiguous areas such as in the Mekong Delta in Vietnam (Becker and Asch, 2005). Single parameter point estimates, however, are of limited usefulness here because it is impossible to vary only one term at a time in equation (4-3).

[30] There is good reason to believe that some important variables in determining the yield are unobserved (e.g. skill level of farmers). I also ran a farmer fixed effects to correct for unaccounted farmer specific factors that may affect the level of fertilizer applied using the data only from farmers' field practice. I only have data on farmers' age and education. I also favored Model 2 over this model because of its greater precision.

4.4.1. Marginal Physical Product and Output Elasticity

Using the regression results reported in Tables 4.2-4.4, I estimated the MPP and output elasticity for each variable on the entire sample plots in all locations. Tables 4.5-4.7 report the MPP and output elasticity for each variable at the mean, along with the associated standard deviations. Except in Thanjavur (TJ) and Uttar Pradesh (UP) India and Can Tho (CT), Vietnam, the MPP of *N* (MPP_N) fertilizer application is positive and the output elasticity is less than one, both significant at 1 percent level. This suggests that additional *N* fertilizer use[31] exerted a significant positive influence on the yield on most plots in the sample. Nitrogen fertilizer application increases the height of the leaves (Chaturvedi, 2005; Mandal et al. 1992), the number of tillers/m^2 (Chaturvedi, 2005; Rajput et al. 1988; Yoshida et al.,1978), and both the number and size of grain (Rupp and Hubner, 1995; Jamieson et al., 1995; Fisher et al., 1977). Note, however, that MPP_N is already decreasing in Aduthurai (AD) (Figure 4.9), Sukamandi (SU) (Figure 4.10), Nueva Ecija (NJ) (Figure 4.11), and Hanoi (HA) (Figure 4.12) but increasing in Suphan Buri, Thailand (SB) at all *N* rates (Figure 4.13). The maximum yield will be achieved at *N* rate where the $MPP_N = 0$. These rates are 139 kg per ha in AD, 135 kg per ha in SU, 160 kg per ha in NJ, and 100 kg per ha in HA. In AD, applying 139 kg per ha of *N* will result to almost 6 tons per ha of grain yield, given all the other factors constant at the mean level. If more than 139 kg per ha is applied, the MPP_N will be negative. This is because excessive *N* promotes lodging and plants become more attractive to insects and diseases.

Meanwhile, the marginal contribution of a kilogram of *P* (MPP_P) is positive and output elasticity is less than one in SU, CT and HA (significant at 1 percent level). The MPP_P is also positive but the output elasticity is greater than one in UP. Phosphorus is a major component in

[31] Henceforth in this section, the term "*N*" refers to "*N* fertilizer applied" and/or "*N* fertilizer." Similar interpretations are used for "*P*" and "*K*."

ATP, the molecule that provides "energy" to the plant for such processes as photosynthesis, protein synthesis, nutrient uptake and nutrient translocation. The magnitude of the estimated coefficients of *P* reveals the significance of this nutrient in rice production, specifically in Vietnam. For example, a kilogram increase in *P* increases yield by 136 kg per ha in HA. Moreover, the MPP_P is increasing at all *P* rates in Vietnam (Figure 4.14 and 4.15).

In contrast, NJ and SB have negative estimated MPP_P and output elasticity at the mean level which are both statistically significant at 1 percent level. There is a possibility that most of the rice straws is retained in the field and hence those soil are often saturated with *P* due to continuous *P* fertilizer application. In fact, the extractable Olsen-P was relatively high on all farms in the sample (IRRI, 2012). If this is the case, no additional amount of *P* fertilizer is required to replenish *P* removed with grain and straw. The additional *P* fertilizer application might result to overapplication. The overapplication of *P* fertilizer does not necessarily lead to environmental damage, but the ability of soil to retain *P* is limited.

Like the *P* fertilizer, the marginal product of *K* fertilizer varies across sites. The marginal contribution of *K* (MPP_K) is positive and output elasticity is less than one in SB. Potassium plays a key role in many metabolic processes in the plant. Proper *K* nutrition in rice promotes (1) tillering, (2) panicle development, (3) spikelet fertility, (4) nutrient uptake of nitrogen and phosphorus, (5) leaf area and leaf longevity, (6) disease resistance, (7) root elongation and thickness, (8) and culm (stem) thickness and strength (Aide and Picker, 1996). A negative MPP_K is observed in two sites in Vietnam. The water from Red River and Mekong River Deltas has high content of sediments, which provides nutrients for crop. Given this, the additional *K* fertilizer application would not be beneficial. If exchangeable *K* and *K* bearing minerals are high in soil, then soil will not be very responsive to *K* fertilizer addition. The rice requirement of *K* is sometimes supplied from plant residues turned under and from *K* in irrigation water (De Datta, 1981).

4.4.2. Evidence of complementarity among N-P-K fertilizers

The main interest of this paper is to explore the relationships among major nutrients and their relationships with inorganic fertilizer, particularly *N* fertilizer to soil fertility, as reflected in soil organic *C* content. Table 4.8 reports the results of the hypotheses testing of the nutrient interactions. The relationship of *N*, *P* and *K* varies across sites. This may be due to the plant's biological processes – *some inputs are complements, some are substitutes, and some are independent*.

The Wald test statistics for the interaction of *N* and *P* are not statistically significant in TJ, UP, NJ, and SB. Given this, one cannot reject the null hypotheses that there is no interaction between *N* and *P,* ($\beta_{NP} = 0$), in the model. The result can be interpreted such that *N* and *P* are independent from each other. If this is the case, the *N* and *P* requirements of crop can be estimated independently and can be applied without the other. However, previous studies report that *N* and *P* are complements (Sheriff, 2005). Nitrogen and *P* are found to be complements in SU. Increasing application of *P* increases the marginal return of *N* (Figure 4.14).[32] Since *P* enhances the root activities of rice crop and when *N* fertilizer is applied to a rice crop that has a healthy, active root system, efficiency is high, because the *N* is absorbed before it can be transformed or lost. Moreover, the movement of *N* within the plant depends largely upon transport through cell membranes, which requires energy to oppose the forces of osmosis. Here, ATP and other high-energy *P* compounds provide the needed energy. In addition, when rice is

[32] The price of paddy rice is set at IDR 3,300 and *N* price is IDR 794.

grown with heavy *N* application, a decline in ratio of filled grains is frequently observed (Mae et al., 2006; Matsushima, 1993) and the only way to further increase the yield is to improve the photosynthesis and biomass production of the rice (Makino, 2011), hence, through *P* fertilizer application.

In this regard, another interpretation for the relationship of *N* and *P* in TJ, UP, NJ, and SB is that *N* might be already limiting in the soil and adding more *P* does not contribute to the crop growth. Typically, the ratio of *N* to *P* is typically lower in manure than needed by crops. This suggests that if farm manure is used to satisfy the *N* requirements of crops, there is a possibility that *P* will be over applied. Given that *N* and *P* are complements, plants require these inputs in a fixed ratio. It is important that SSNM accounts for the proper input ratio in its algorithm. Based on Tables 4.5 and 4.6, the MPP_P is statistically insignificant in TJ and NE. If *N* is indeed limiting in the soil, then yields will be unresponsive to increases in *P*, so $\beta_{NP} = 0$.

There is also no significant interaction between *P* and *K,* ($\beta_{PK} = 0$), in AD, UP, NJ, CT, and HA. Again, it is possible that one of the nutrients is already limiting. The marginal products of *P* and *K* are very low or even negative. If farmers have been practicing selective fertilizer application, i.e. only applying *N*, applying *P* to the soil will be limiting in the long run. Adding more *K* will have no indirect effect on yield. The Wald test also fails to reject the null hypothesis that there is no interaction between *N* and *K* fertilizer in AD, TJ, SU, NJ, CT, and HA.

While we expect that *N* and *P* are complements, interestingly, results of my study provide clear evidence of substitution between *N* and *P* in AD, CT, and HA. Figures 4.15 – 4.17 display the kernel-weighted local polynomial smoothing of the estimated marginal value product of *N* against *P*, along with the cost of *N* and *P* fertilizer inputs (red horizontal line)[33]. The figures suggest that if two nutrients are substitutes, increasing the application of one nutrient reduces the marginal returns of the other nutrient. The marginal returns of *N* are all higher than the cost of *N* at almost all levels of *P* in AD and HA. In CT, beyond 20 kg/ha of *P* applied, the marginal returns to *N* are less than the price of *N*. In light of increasing fertilizer costs and the fact that currently known phosphate rock reserves, the source of *P* fertilizer, are finite, it is indeed important that SSNM fertilizer algorithm accounts not only for this substitution relationship of *N* and *P* but also for the input costs in its recommendation to make better use of applied- and soil-endogenous *N* and *P*. This finding can also support the practice of farmers of selective application of nutrients. Compared to phosphate and potash fertilizer, *N* fertilizer is heavily subsidized in India. Hence, this adversely affects the consumption of *P* and *K* fertilizer. While the substitution of *N* and *P* maybe justified on economic grounds, this relationship needs further research or studies that support it from a biological viewpoint.

On the other hand, *N* and *K* are found to be complements in UP. Like *P, K* plays an important role in physiological process of rice, and contributes to greater canopy photosynthesis and crop growth. Potassium also increases the number of spikelets per panicle (flowers per grain bunch) and the percentage of filled grain. Although *N* and *K* are also complements in UP, *N* and *K* significantly decrease yield, $\beta_N < 0$ and $\beta_K < 0$ while significantly increasing the marginal product of *N*, ($\beta_{NK} > 0$) (Table 4.2). This suggests that a positive relationship between yield and *K* can occur only if the positive effect of *K* on the marginal product of *N* is higher in absolute value than is the direct effect of *K* on yield. This also indicates that *K* must not be applied alone, but rather in combination with *N*. Given this, selective application of fertilizer, i.e. only applying

[33] A kilo of rice is INR 24 in India and VND 8,000 in Vietnam. The input cost of *N* fertilizer INR 30 in India and VND 5,600 in Vietnam.

N or K when farmers are faced with cash constraints, might bring more harm than good to the crop.

In Thailand, the resulting estimates are intuitive but quite inconsistent. Nitrogen fertilizer can be substituted for P ($\beta_{NP} < 0$), and P can be substituted for K ($\beta_{PK} < 0$). Hence, by transitivity, N and K are substitutes ($\beta_{NK} < 0$) as well. Interestingly, results suggest otherwise. N and K are complements $\left(\beta_{NK} > 0\right)$, and is statistically significant (at 5 percent level) implying an increase yield due to the positive effect of K on the marginal product of N (Table 4.3). All else held constant, an extra kilogram of K is associated with an almost two-kilogram increase in yield to a kilogram of N. Given that K is not usually applied in Thailand, deficiency of K will not be problem because nearly all rice straw (which is high in K) is left on the ground after harvest (Moya et al. 2004). *In general, results discussed above suggest that nutrient interaction matters in making fertilizer recommendations to farmers and hence the SSNM strategy should explicitly account for these nutrient interactions in its algorithm to ensure effectiveness of fertilizer application.*

4.4.3. Is yield response to N fertilizer dependent on the ex-ante state of soil?

I also hypothesized that the yield response to N fertilizer is dependent on the ex-ante state of soil condition. At the ten percent significance level, the Wald test rejects the hypothesis that the interaction term of N and soil C content is jointly zero, $\beta_{OrgCN} = 0$, implying complementarity between soil fertility and N in Indonesia, Philippines and Hanoi, Vietnam (Table 4.8).

Figures 4.18-4.20 display the kernel-weighted local polynomial smoothing of the estimated marginal value product (MVP) of N against the plots' soil organic C contents for SU, NJ, and HA. The figures clearly provide evidence that there exists a positive relationship between fertilizer yield response and soil *C* content. The marginal returns to *N* exceed the price of *N* fertilizer in all three locations at all levels of a plot's soil organic content. The *N* fertilizer price is IDR 794 per kilogram, PHP 13.30 per kilogram and VND 5600 per kilogram in SU, NJ, and HA, respectively. Figure 4.18 displays that the marginal returns to *N* in Indonesia are do not vary up to a *C* content level of approximately 13 g/kg, at which point the marginal returns to *N* increase up to a *C* content level of approximately 21 g/kg, after which the MVP of *N* nearly plateaus beyond that *C* content level. This is consistent with previous findings of Marenya and Barrett (2009) in Western Kenyan farms. They observed an S-shaped relation between fertilizer yield response and soil *C* stocks. Figure 4.19, on the other hand, suggests that the MVP of *N* is rapidly increasing in all Philippine sample plots. Figure 4.20 shows that up to a *C* content level of approximately 17 g/kg, the marginal returns to *N* in Hanoi do not vary, then it increases at an increasing rate up to a *C* content level of approximately 22 g/kg, after which it increases at a decreasing rate. These results suggest that if further investments are devoted to increasing soil *C* content in Vietnam, *N* fertilizer application is expected to be profitable.

Although the Wald test failed to reject the hypothesis that the interaction term of *N* and soil carbon content *C* are jointly zero, $\beta_{OrgCN} = 0$, in all three sites in India, CT and SB, it is possible that soil *C* content in these areas is already limiting and adding more *N* does not contribute to the crop growth. For example, Figure 4.21 shows that at more than around 8 g/kg carbon content, the marginal returns of *N* fertilizer start to increase in AD. On average, the soil *C* content in AD is only 9 g/kg.

Government interventions such as fertilizer provision or subsidies might not be effective in raising farmers' yield, and eventually profit, where soil organic matter is already a limiting input. The yield response of rice to *N* depends on the initial state of the soil and hence ex-ante soil conditions must be explicitly accounted for in the SSNM algorithm. Although IRRI scientists strongly encourage farmers to use organic fertilizer such as farmyard manure in their rice fields, this does not discount the need to explicitly incorporate the interaction of soil *C* content and *N* in SSNM algorithm. In order for farmers to reap significant economic returns from *N* fertilizer application, soil scientists must ensure that there is adequate amount of soil organic matter.

4.4.4. Non-nested Hypothesis Test Results

The current SSNM algorithm, which uses the yield goal approach, only makes economic sense if the crop production function is linear von Liebig. The results of the non-nested hypothesis tests in rejected the linear von Liebig model specification, except in AD (Table 4.9). The quadratic model outperformed all the rival specifications, both in a pairwise comparison as well as in a collective test against all the alternatives. In this regard, the yield goal-based approach in SSNM strategy can be misleading. In the case of a quadratic functional form, profit maximization requires information on input and output prices and the marginal product of each increment of fertilizer. The economic optimal fertilizer rate is attained when the marginal product of fertilizer is equal to the ratio of input and output price. Given a non-zero price ratio, there is a difference between the yield maximizing and profit maximizing input levels. Rising fertilizer prices are a particular problem for poor farmers who could not afford sufficient fertilizers.

4.5. Conclusion and Policy Implications

In this study, I have reported clear evidence that interaction among major nutrients matters in making fertilizer recommendations to farmers. The relationships among *N*, *P*, and *K* vary across sites -- *some inputs are complements, some are substitutes, and some are independent*. I also found that soil organic matter, manifested in soil *C* stocks, significantly affected the economic returns to *N* fertilizer inputs. The marginal product on *N* is low on soils with low *C* content. These results suggest the SSNM strategy should explicitly account for the: (1) nutrient interactions and (2) relationship of *N* fertilizer and soil organic matter, as reflected in soil *C* stocks. Accounting for these effects will make the SSNM strategy more adaptive to farmers' fields and will allow the integration of nutrient management techniques for maximum benefit to rice producers. The application of essential plant nutrients, particularly major nutrients and soil organic matter, in optimal quantities and proportions is the key to increased and sustained rice production. In addition, input and output prices should also not be ignored in SSNM algorithm. The quadratic model specification of the crop response outperformed linear von Liebig model. The major challenge for SSNM will be to retain the simplicity of the approach that is understandable to producers and extension agents while accounting for the relationship of *NPK* and soil organic matter.

The results of this study could stimulate not only IRRI scientists but also policymakers to review the existing fertilizer policies in the sample countries, the path that these countries choose to take on fertilizer policy has significant implications for food security through the global market for rice. While the decisions of farmers about fertilizer use depend upon which fertilizers are cheaper to obtain and apply, government should not only focus on policies conducive to increased availability and consumption of fertilizers. To ensure the effectiveness of fertilizer policies, they must be targeted not only to match the needs, preferences, and resources of farmers, but also to account for the interactions of production inputs. The substitutability and complementarity of major nutrients and soil organic matter are also critical in the decision-making processes of policymakers from the sample countries. The blending of fertilizer should be tailored to the relationship of nutrients to be mixed.

If major nutrients are complements, then direct subsidies for these nutrients must be provided. For example, if N and P are complements, low subsidized prices for N fertilizer matched by similar level for P fertilizer reduces the probability of farmers practicing selective application when they are faced with cash constraints. Fertilizer subsidy or distribution might also not be appropriate means to support rice production, however, in areas where soils have limiting organic matter content. The initial soil conditions matter to the return on investments in fertilizer policies. In such a case, government intervention should also consider putting greater emphasis on integrated soil fertility management and adoption of soil conservation technologies. Organic sources of nutrients (e.g. farmyard manure, crop residues carried over) must not only be promoted as a response to rising prices of commercial manufactured fertilizers but also as a basis for increasing productivity. Extension agencies and others can potentially encourage further adoption of the use of organic fertilizers by emphasizing to farmers the benefit of organic materials on the physical properties of rice soils.

While the results of this paper suggest that nutrient interactions among major nutrients and soil organic matter tend to vary from site to site, there are two caveats to keep in mind when interpreting these results. First, while the economic analysis suggest that N and P are substitutes, this relationship needs further research or studies that support it from a biological viewpoint. Most of the previous studies suggest that N and P are complementary inputs. The second caveat is that results from this study only pertain to one to two years of experiment. If the crop response function to major nutrients and soil organic matter varies from year to year, the results are only representative for a given state of nature observed at certain point in time (Anselin, Bongiovanni, and Lowenberg-DeBoer, 2004). A multi-year analysis would be an interesting extension of this study. This demonstrates a frontier where agricultural economists and agronomists can work together.

Table 4.1a. Study area, RTDP, IRRI

COUNTRY	REGION/ PROVINCE	RICE DOMAIN	NO. OF FARMERS	CROPPING SYSTEM	CLIMATE	YEARS INCLUDED	CROPPING SEASON[a]
India	Tamil Nadu	Aduthurai	40	Rice-rice	Tropical	97	KR, TH
		Thanjavur	19	Rice-rice	Tropical	97,99	KR, TH
	Uttar Pradesh	Pantnagar	23	Rice-wheat	Sub-Tropical	97	KH
Indonesia	West Java	Sukamandi	30	Rice-rice	Tropical	96,98	DS, WS
Philippines	Nueva Ecija	Maligaya	50	Rice-rice	Tropical	95-96	DS, WS
Thailand	Central Plain	Suphan Buri	27	Rice-rice	Tropical	95-96	DS, WS
Vietnam	Mekong Delta	Can Tho	32	Rice-rice-rice	Tropical	96	DS, WS
	Red River Delta	Hanoi	24	Rice-rice-maize	Sub-Tropical	97	ER, LR

[a] *High yielding season: KR - Kuruvai, DS – Dry Season, ER – Early Rice; Low yielding season: TH – Thaladi, WS – Wet Season, LR – Late Rice.*

Table 4.1b. Description of variables

Variable	Description
Rice output (kg/ha)	Dependent variable. Kilograms of rice harvested per season in a given year
Nitrogen applied (N)	Kilogram of nitrogen (N) from fertilizers applied
Phosphorus applied (P)	Kilogram of phosphorus (P) from fertilizers applied
Potassium (K)	Kilogram of potassium (K) from fertilizers applied
Org C	Amount of carbon content in the soil (g/kg)
Age (year)	Age in years of the person responsible for production decisions on the plot
Educ (year)	Total years of schooling completed by the farmer
Farm area (ha)	Size of farm owned by the farmer
High yielding season (HYS)	Dummy variable. HYS=1; high yielding season HYS=0; low yielding season

Table 4.1c. Descriptive statistics

Site/Variable	Obs	Mean	Std. Dev.	Min	Max
INDIA					
Aduthurai					
Rice output (kg/ha)	1121	5,128.03	1,454.71	1,125.00	9,325.00
N applied (kg/ha)	1121	52.87	64.90	0.00	222.97
P applied (kg/ha)	1121	17.54	14.41	0.00	54.58
K applied (kg/ha)	1121	32.95	30.87	0.00	163.47
Org C (g/kg)	1121	9.04	1.25	4.50	14.90
Age (year)	867	47.31	11.74	26.00	70.00
Educ (year)	274	10.58	2.84	5.00	18.00
Farm area (ha)	1121	0.30	0.08	0.00	0.54
HYS	1121	0.37	0.48	0.00	1.00
Thanjavur					
Rice output (kg/ha)	77	4,632.96	1,281.16	1,710.00	7,629.00
N applied (kg/ha)	77	48.34	56.06	0.00	253.10
P applied (kg/ha)	77	10.60	15.31	0.00	72.47
K applied (kg/ha)	77	20.53	30.05	0.00	125.40
Org C (g/kg)	77	71.15	7.88	56.00	85.00
Age (year)	-	-	-	-	-
Educ (year)	-	-	-	-	-
Farm area (ha)	75	0.31	0.17	0.16	0.93
HYS	77	0.92	0.27	0.00	1.00
Uttar Pradesh					
Rice output (kg/ha)	84	5,068.41	1,190.91	2,361.00	7,648.00
N applied (kg/ha)	84	62.97	72.61	0.00	252.50
P applied (kg/ha)	84	24.64	8.44	3.18	51.35
K applied (kg/ha)	84	30.05	21.00	0.00	50.00
Org C (g/kg)	84	11.89	2.71	4.55	16.50
Age (year)	80	50.35	11.60	30.00	74.00
Educ (year)	40	11.10	3.37	5.00	16.00
Farm area (ha)	84	0.36	0.08	0.10	0.40
HYS	84	0.00	0.00	0.00	0.00

Table 4.1c. Continued…

Site/Variable	Obs	Mean	Std. Dev.	Min	Max
INDONESIA					
Sukamandi, West Java					
Rice output (kg/ha)	480	4,046.43	1,372.89	539.00	7,727.00
N applied (kg/ha)	480	55.36	66.03	0.00	253.97
P applied (kg/ha)	480	11.24	12.77	0.00	36.59
K applied (kg/ha)	480	17.37	23.83	0.00	102.12
Org C (g/kg)	480	15.70	4.97	7.93	24.90
Age (year)	435	43.30	13.81	24.00	82.00
Educ (year)	142	6.92	3.28	1.00	12.00
Farm area (ha)	480	0.99	1.18	0.10	5.33
HYS	480	0.78	0.42	0.00	1.00
PHILIPPINES					
Nueva Ecija, Philippines					
Rice output (kg/ha)	630	4,760.10	1,559.10	907.00	9,922.00
N applied (kg/ha)	630	41.96	63.98	0.00	266.15
P applied (kg/ha)	630	13.79	12.89	0.00	32.18
K applied (kg/ha)	630	22.83	22.11	0.00	61.80
Org C (g/kg)	630	10.39	2.78	4.02	16.50
Age (year)	558	51.02	13.60	24.00	84.00
Educ (year)	179	7.32	4.03	0.00	14.00
Farm area (ha)	630	1.73	0.96	0.40	5.00
HYS	630	1.00	0.00	1.00	1.00
THAILAND					
Suphan Buri, Thailand					
Rice output (kg/ha)	660	3,572.47	960.24	1,173.00	6,615.00
N applied (kg/ha)	660	34.61	52.66	0.00	191.99
P applied (kg/ha)	660	17.13	13.99	0.00	53.85
K applied (kg/ha)	660	16.69	23.52	0.00	50.00
Org C (g/kg)	660	10.49	6.67	0.78	25.14
Age (year)	651	46.91	8.84	28.00	70.00
Educ (year)	216	4.78	1.85	2.00	10.00
Farm area (ha)	660	1.55	0.96	0.16	3.52
HYS	660	0.65	0.48	0.00	1.00

Table 4.1c. Continued…

Site/Variable	Obs	Mean	Std. Dev.	Min	Max
VIETNAM					
Can Tho, Vietnam					
Rice output (kg/ha)	591	3,894.34	1,415.28	743.00	7,608.00
N applied (kg/ha)	591	32.22	54.18	0.00	182.21
P applied (kg/ha)	591	15.38	13.82	0.00	51.19
K applied (kg/ha)	591	19.20	22.38	0.00	50.00
Org C (g/kg)	591	18.54	4.11	10.80	31.70
Age (year)	591	47.80	11.00	30.00	67.00
Educ (year)	591	6.86	3.65	1.00	12.00
Farm area (ha)	591	0.81	0.67	0.00	3.60
HYS	591	0.65	0.48	0.00	1.00
Ha Noi, Vietnam					
Rice output (kg/ha)	96	5,627.50	1,389.42	2,840.00	9,975.00
N applied (kg/ha)	96	48.12	50.67	0.00	143.75
P applied (kg/ha)	96	24.25	8.10	6.01	36.42
K applied (kg/ha)	96	51.05	14.71	0.00	97.65
Org C (g/kg)	96	14.74	4.98	7.50	24.50
Age (year)	48	47.75	9.15	32.00	63.00
Educ (year)	24	7.08	2.65	2.00	10.00
Farm area (ha)	96	0.08	0.02	0.06	0.15
HYS	96	0.96	0.20	0.00	1.00

Table 4.2. Quadratic Rice Production Function Estimates, India

Variable	Aduthurai		Uttar Pradesh		Thankjavur	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
N	25.59***	25.30***	-128.7***	-107.2***	-9.85	-12.70
	(5.62)	(5.63)	(36.95)	(35.31)	(29.23)	(30.46)
Nsq	-0.107***	-0.107***	0.378**	0.366**	-0.01	0.04
	(0.01)	(0.01)	(0.16)	(0.15)	(0.11)	(0.12)
P	-19.97	-17.67	1024.5***	855.8**	24.54	84.46
	(15.70)	(15.78)	(345.60)	(328.70)	(99.92)	(120.90)
Psq	0.956***	0.870**	-20.86	-14.18	0.04	0.35
	(0.36)	(0.37)	(14.61)	(13.94)	(0.65)	(0.72)
K	5.74	6.37	-1054.7**	-840.8**	53.90**	34.73
	(7.14)	(7.18)	(426.70)	(406.50)	(25.57)	(32.41)
Ksq	-0.114***	-0.111***	2.72	2.87	-0.12	-0.03
	(0.04)	(0.04)	(2.45)	(2.31)	(0.16)	(0.18)
NP	-0.221*	-0.214*	-1.34	-1.91	-0.05	-0.70
	(0.12)	(0.12)	(2.17)	(2.09)	(1.18)	(1.38)
PK	-0.03	-0.05	19.93	12.39	-1.11	-1.474*
	(0.15)	(0.15)	(17.24)	(16.41)	(0.74)	(0.84)
NK	0.08	0.07	4.566***	4.102***	-0.14	0.07
	(0.05)	(0.05)	(1.21)	(1.15)	(0.26)	(0.35)
OrgC	173.50	175.90	-149.30	-286.70	882.2***	785.4***
	(217.40)	(218.40)	(200.50)	(209.50)	(233.30)	(261.80)
OrgCsq	-7.56	-8.08	5.51	10.39	-5.941***	-5.296***
	(11.61)	(11.67)	(8.89)	(9.67)	(1.62)	(1.81)
OrgCN	0.62	0.63	0.54	0.49	0.28	0.24
	(0.46)	(0.46)	(0.63)	(0.60)	(0.29)	(0.32)
HYS		138.1*		-		326.20
		(71.63)		-		(588.40)
Farm area		2,250.20		-21061.1**		448.50
		(2,518.30)		(8,018.80)		(2,741.90)
Farm area x farm area		-2,860.50		40513.2***		359.80
		(4,017.70)		(13,753.90)		(2,441.70)
Constant	3,310.8***	2,879.2**	9,308.7***	11,624.0***	2,858.2***	2,550.6***
	(1,013.80)	(1,120.10)	(1,756.90)	(1,900.30)	(8,317.30)	(9,128.20)
No. of observations	1,121	1,121	84	84	77	75
Adjusted R-squared	0.41	0.41	0.51	0.57	0.64	0.63
Akaike Info Criteria	18,934.80	18,935.50	1,380.16	1,371.33	1,253.15	1,226.73
Bayesian Info Criteria	19,000.09	19,015.85	1,411.76	1,407.80	1,283.62	1,263.81

Standard errors in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 4.3. Quadratic Rice Production Function Estimates, Indonesia, Philippines, and Thailand

Variable	West Java, Indonesia		Nueva Ecija, Phillippines		Suphan Buri, Thailand	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
N	21.32***	24.11***	17.43***	19.05***	-7.73	-4.57
	(3.73)	(3.21)	(6.29)	(6.30)	(5.87)	(5.69)
Nsq	-0.0881***	-0.0822***	-0.0662***	-0.0701***	0.0864**	0.06
	(0.01)	(0.01)	(0.02)	(0.02)	(0.04)	(0.04)
P	-10.75	8.97	22.36	25.85	78.79***	74.12***
	(24.04)	(20.54)	(59.25)	(59.16)	(24.87)	(24.06)
Psq	0.45	-0.47	-1.05	-1.19	-0.48	-0.45
	(0.76)	(0.65)	(2.56)	(2.55)	(0.48)	(0.46)
K	-40.64***	-19.55	-9.66	-13.03	-	-14.42
	(14.04)	(12.06)	(29.42)	(29.77)	-	(13.62)
Ksq	0.395***	0.198*	0.841*	0.851*	7.28	7.653*
	(0.13)	(0.11)	(0.46)	(0.45)	(4.67)	(4.56)
NP	0.255*	0.16	0.10	0.07	-0.381*	-0.33
	(0.13)	(0.11)	(0.33)	(0.33)	(0.23)	(0.23)
PK	0.844**	0.493*	-0.84	-0.73	-13.41*	-13.96*
	(0.33)	(0.28)	(1.41)	(1.41)	(7.78)	(7.58)
NK	0.05	0.02	-0.15	-0.16	1.787**	1.748**
	(0.09)	(0.07)	(0.19)	(0.19)	(0.89)	(0.87)
OrgC	91.64	73.59	74.54	137.90	-17.47	29.82
	(72.39)	(63.72)	(124.20)	(126.30)	(19.83)	(28.79)
OrgCsq	2.32	1.48	-3.10	-5.42	-0.01	-1.71
	(2.21)	(1.93)	(5.71)	(5.77)	(0.98)	(1.22)
OrgCN	-0.10	-0.273**	0.684**	0.631*	0.08	0.04
	(0.16)	(0.14)	(0.34)	(0.34)	(0.10)	(0.09)
HYS		1402.6***		-		533.7***
		(109.30)		-		(127.80)
Farm area		620.2***		-474.5**		-8.54
		(129.00)		(187.50)		(202.30)
Farm area x farm area		-122.3***		103.1**		-0.91
		(24.54)		(43.77)		(48.16)
Constant	1,338.7**	449.10	3,719.0***	3,744.8***	3,471.4***	2,901.3***
	(562.70)	(494.80)	(660.10)	(659.90)	(96.53)	(355.60)
No. of observations	480	480	630	630	660	660
Adjusted R-squared	0.47	0.62	0.29	0.29	0.20	0.26
Akaike Info Criteria	8,006.94	7,855.51	10,851.17	10,848.61	10,802.13	10,757.67
Bayesian Info Criteria	8,061.20	7,922.29	10,908.97	10,915.29	10,856.04	10,825.05

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

Table 4.4. Quadratic Rice Production Function Estimates, Vietnam

Variable	Can Tho		Ha Noi	
	Model 1	Model 2	Model 1	Model 2
N	-8.63	16.94**	83.23	69.25
	(14.60)	(8.55)	(65.47)	(63.38)
Nsq	0.06	0.03	-0.22	-0.256*
	(0.09)	(0.06)	(0.14)	(0.14)
P	235.8***	57.89	231.90	204.30
	(83.93)	(49.16)	(165.80)	(164.40)
Psq	-0.53	2.128**	1.46	2.62
	(1.51)	(0.88)	(2.66)	(2.61)
K	-38.39	-17.87	-53.79	-139.40
	(81.27)	(47.03)	(140.90)	(139.40)
Ksq	0.26	-1.33	0.38	0.74
	(1.67)	(0.97)	(0.58)	(0.58)
NP	-1.15	-1.209**	-2.550**	-2.741***
	(0.84)	(0.49)	(1.00)	(1.00)
PK	-3.41	0.58	-0.40	-0.18
	(3.23)	(1.88)	(1.51)	(1.53)
NK	0.86	0.23	0.18	0.64
	(0.54)	(0.31)	(0.88)	(0.87)
OrgC	-521.1***	-446.2***	-205.10	-265.40
	(109.30)	(62.95)	(156.50)	(160.20)
OrgCsq	12.61***	9.917***	11.01**	12.61**
	(2.76)	(1.59)	(4.80)	(4.83)
OrgCN	-0.21	-0.20	1.168**	1.224**
	(0.28)	(0.16)	(0.53)	(0.51)
HYS		2232.2***		1122.1*
		(68.55)		(568.40)
Farm area		507.6***		-73770.9**
		(141.20)		(35,356.70)
Farm area x farm area		-103.3**		379887.0**
		(45.94)		(178,261.70)
Constant	8,650.2***	6,384.1***	341.90	5,167.60
	(1,052.80)	(616.50)	(7,097.70)	(7,221.50)
No. of observations	591	591	96	96
Adjusted R-squared	0.14	0.72	0.50	0.54
Akaike Info Criteria	10,175.29	9,520.39	1,607.71	1,602.75
Bayesian Info Criteria	10,232.26	9,590.50	1,641.05	1,643.78

Standard errors in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 4.5. Marginal physical product (MPP) and output elasticity at the mean, India

Variable	Aduthurai		Thanjavur		Uttar Pradesh	
	MPP	Output Elasticity	MPP	Output Elasticity	MPP	Output Elasticity
Total N (kg)	18.34 (1.36)***	0.19 (0.02)***	2.01 (12.85)	0.02 (0.13)	20.77 (22.67)	-0.09 (0.17)
Total P (kg)	-0.04 (6.59)	-0.01 (0.02)	27.77 (58.08)	0.06 (0.13)	408.94 (196.77)**	1.20 (0.87)**
Total K (Kg)	2.05 (3.97)	0.02 (0.02)	21.25 (16.9)	0.09 (0.07)	-104.99 (80.55)	-1.17 (0.49)**
Org C (g/kg)	63.06 (30.72)**	0.12 (0.05)**	43.32 (13.75)***	0.66 (0.21)***	-9.16 (47.81)	-0.02 (0.13)
Farm area	533.90 (472.02)	0.02 (0.03)	685.97 (1467.44)	0.04 (0.10)	918.68 (302.20)***	0.77 (0.25)***

Standard deviations in parenthesis
* p<0.10, ** p<0.05, *** p<0.01

Table 4.6. Marginal physical product (MPP) and output elasticiy at the mean, Indonesia, Philippines, and Thailand

Variable	Sukamandi, West Java, Indonesia		Nueva Ecija, Philippines		Suphan Buri, Thailand	
	MPP	Output	MPP	Output	MPP	Output
Total N (kg)	12.96 (1.04)***	0.18 (0.01)***	17.10 (2.43)***	0.15 (0.02)***	36.27 (8.27)***	0.32 (0.08)***
Total P (kg)	15.94 (7.68)**	0.04 (0.02)**	-20.53 (21.89)	-0.06 (0.06)	-270.83 (64.42)***	-0.70 (0.17)***
Total K (Kg)	-5.86 (5.06)	-0.02 (0.02)	9.15 (13.36)	0.04 (0.06)	230.31 (54.44)***	0.48 (0.12)***
Org C (g/kg)	104.96 (12.02)***	0.41 (0.05)***	51.82 (20.87)**	0.11 (0.05)**	-3.98 (8.17)	-0.01 (0.02)
Farm area	380.40 (94.06)***	0.09 (0.02)***	-123.87 (63.82)*	-0.04 (0.02)*	-11.78 (58.89)	-0.01 (0.03)

Standard deviations in parenthesis
* p<0.10, ** p<0.05, *** p<0.01

Table 4.7. Marginal physical product (MPP) and output elasticity at the mean, Vietnam

Variable	Can Tho		Ha Noi	
	MPP	Output Elasticity	MPP	Output Elasticity
Total N (kg)	1.36 (0.82)	0.01 (0.05)	28.81 (6.30)***	0.24 (0.05)***
Total P (kg)	95.54 (24.34)***	0.38 (0.10)***	190.42 (55.69)***	0.82 (0.24)***
Total K (Kg)	-52.44 (15.45)***	-0.26 (0.08)***	-37.11 (46.42)	-0.33 (0.42)
Org C (g/kg)	-84.76 (7.59)***	-0.40 (0.04)***	165.18 (32.24)***	0.43 (0.08)***
Farm area	340.30 (80.21)***	0.07 (0.02)***	-575.38 (244.84)**	-0.04 (0.03)*

Standard deviations in parenthesis
* $p<0.10$, ** $p<0.05$, *** $p<0.01$

Table 4.8. Results of Hypothesis Testing, p-value.

Hypothesis: Parameter β_{ij}	Aduthurai, India	Thanjavur, India	Uttar Pradesh, India	Sukamandi, WJ, Indonesia	Nueva Ecija, Philippines	Suphan Buri, Thailand	Can Tho, Vietnam	Hanoi, Vietnam
NP = 0	0.09	0.61	0.36	0.14	0.82	0.13	0.00	0.00
NP < 0	0.95	0.69	0.82	0.07	0.41	0.93	0.99	0.99
NP > 0	0.04	0.31	0.18	0.93	0.59	0.07	0.01	0.01
PK = 0	0.73	0.08	0.45	0.08	0.61	0.06	0.70	0.88
PK < 0	0.64	0.95	0.23	0.04	0.70	0.97	0.35	0.55
PK > 0	0.36	0.05	0.77	0.96	0.30	0.03	0.65	0.45
NK = 0	0.10	0.84	0.00	0.76	0.41	0.04	0.92	0.49
NK < 0	0.05	0.42	0.00	0.38	0.79	0.02	0.17	0.25
NK > 0	0.95	0.58	1.00	0.62	0.21	0.98	0.83	0.75
OrgCN = 0	0.18	0.45	0.42	0.04	0.06	0.65	0.15	0.08
OrgCN < 0	0.09	0.23	0.21	0.98	0.03	0.33	0.93	0.04
OrgCN > 0	0.91	0.77	0.79	0.02	0.97	0.67	0.07	0.96

Table 4.9. Nonnested hypothesis results based on P_0 test

SITE/ ALTERNATIVE HYPOTHESIS	NULL HYPOTHESIS			
	Linear von Liebig	Squared	Square-root	nonlinear von Liebig
India				
Aduthurai				
Linear von Liebig	-	12.41***	1.04	10.88**
Squared	1.12	-	0.03	1.66
Square-root	0.78	9.84***	-	1.7
nonlinear von Liebig	21.57***	13.74***	3.81*	-
ALL	3.10**	6.59***	2.91**	0.64
Thanjavur				
Linear von Liebig	-	1.81	1.83	2.09
Squared	5.11**	-	0.94	3.14*
Square-root	4.40**	0	-	3.70*
nonlinear von Liebig	69.16***	4.13**	7.48**	-
ALL	2.44*	1.34	2.04	1.33
Uttar Pradesh				
Linear von Liebig		0.15	1.87	0.8
Squared	3.84*		0.48	11.61***
Square-root	3.47*	0.12		12.50***
nonlinear von Liebig	6.94***	0.26	0.83	
ALL	1.54	0.12	1.09	4.35***
West Java, Indonesia				
Linear von Liebig	-	0.1	2.46	89.54***
Squared	53.74***	-	2.85*	268.64
Square-root	58.63***	3.47*	-	14.80***
nonlinear von Liebig	51.36***	0.68	0.06	260.41***
ALL	28.56***	1.94	2.58*	91.48***
Nueva Ecija, Philippines				
Linear von Liebig		0.05	0.69	3.22*
Squared	23.17***		2.86*	0.47
Square-root	25.66***	3.18*		0.7
nonlinear von Liebig	49.01***	2.01	2.49	
ALL	15.97***	7.04***	8.31***	0.67

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

Table 4.9. Continued….

SITE/ ALTERNATIVE HYPOTHESIS	NULL HYPOTHESIS			
	Linear von Liebig	Squared	Square-root	nonlinear von Liebig
Suphan Buri, Thailand				
Linear von Liebig		0.01	22.57***	66.44***
Squared	24.70***		26.04***	17.24***
Square-root	28.87***	6.66**		17.81***
nonlinear von Liebig	10.21***	0.71	21.44***	
ALL		2.52*	18.31***	6.09***
Vietnam				
Can Tho				
Linear von Liebig	-	0.22	2	6.45*
Squared	21.48***	-	7.62***	7.91***
Square-root	15.49***	0.01	-	1.65
nonlinear von Liebig	27.05***	2.04	9.87***	-
ALL	10.31***	1.98	6.46***	4.55***
Hanoi				
Linear von Liebig	-	0.13	6.41**	0.16
Squared	6.20**	-	0.08	3.8*
Square-root	5.46**	6.37**	-	3.30*
nonlinear von Liebig	17.08***	0.67	5.21**	-
ALL	4.88***	1.45	2.35*	4.08***

Standard errors in parentheses
* p<0.10, ** p<0.05, *** p<0.01

Figure 4.1. Rice yield as a function of plot's carbon content
Aduthurai, India

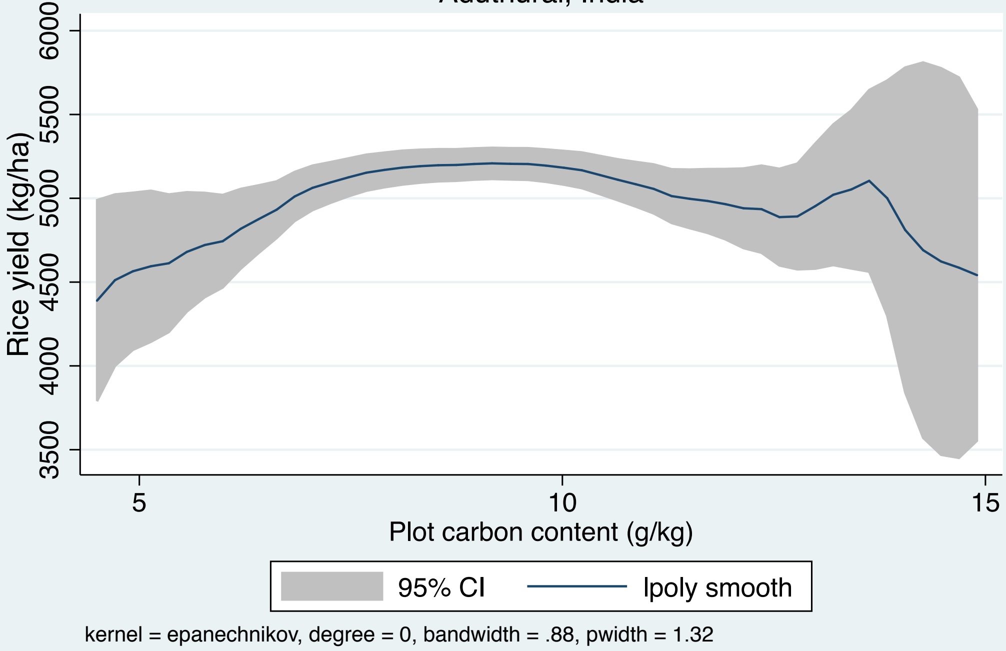
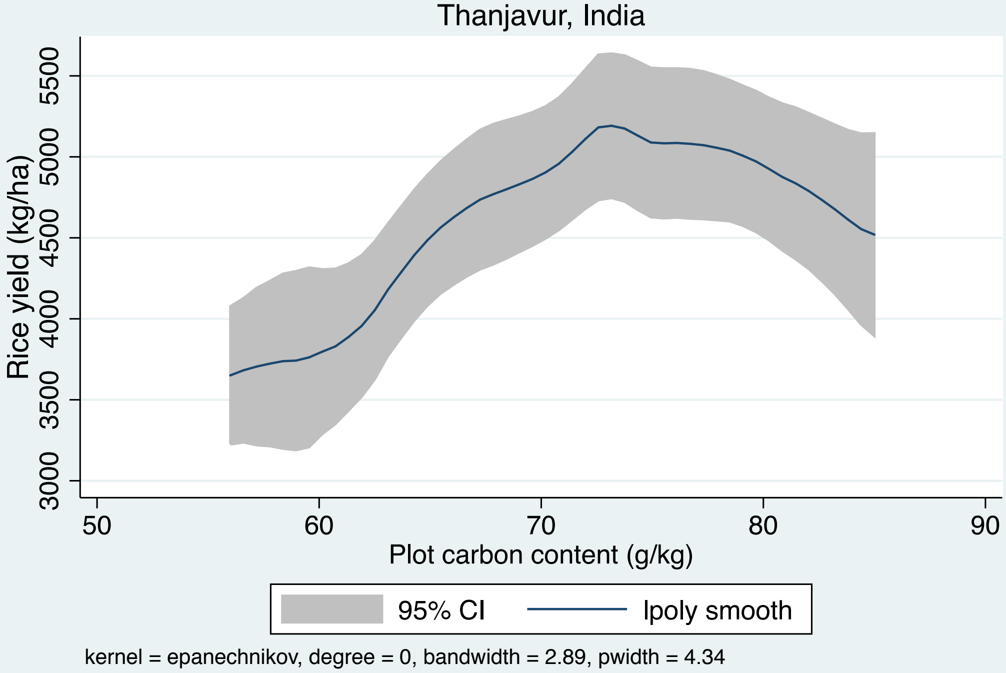


Figure 4.2. Rice yield as a function of plot's carbon content
Thanjavur, India



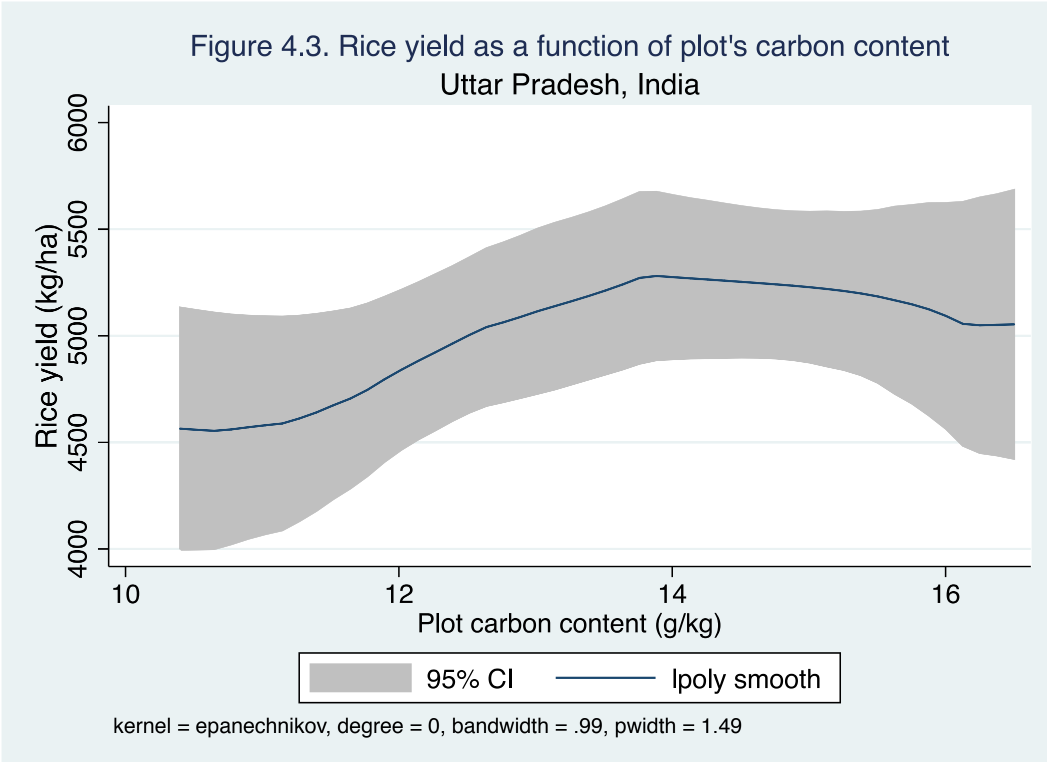


Figure 4.3. Rice yield as a function of plot's carbon content
Uttar Pradesh, India
Rice yield (kg/ha)
4000
4500
5000
5500
6000
10
12
14
16
Plot carbon content (g/kg)
95% CI
lpoly smooth
kernel = epanechnikov, degree = 0, bandwidth = .99, pwidth = 1.49

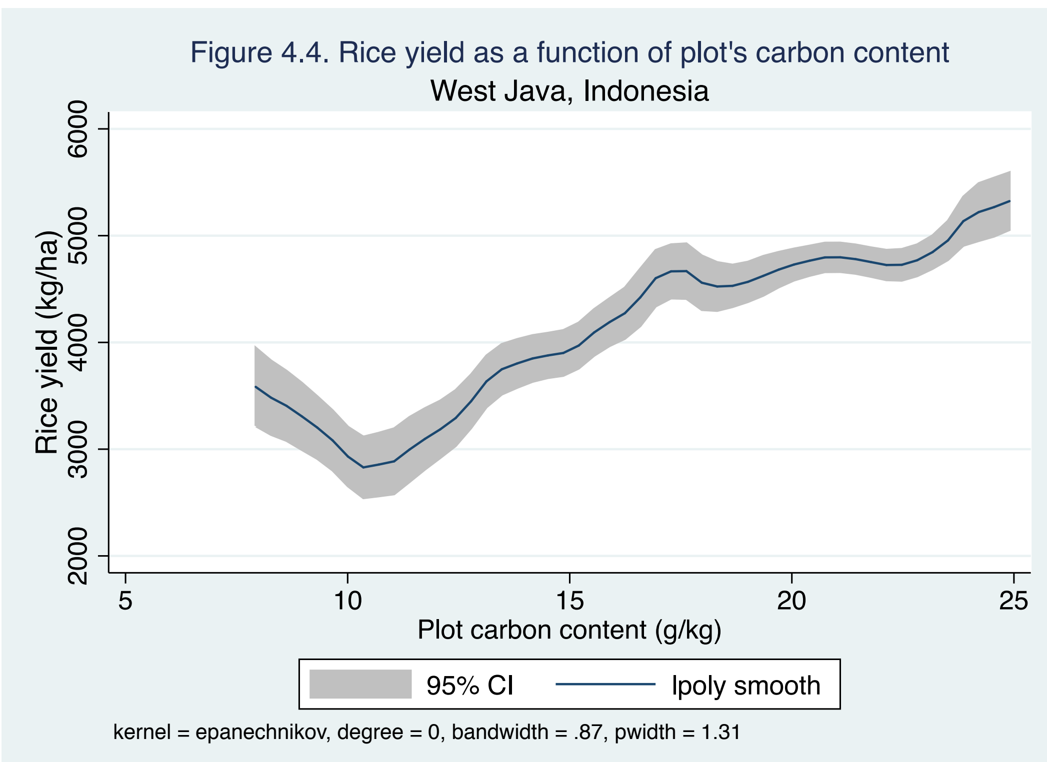


Figure 4.4. Rice yield as a function of plot's carbon content
West Java, Indonesia
Rice yield (kg/ha)
2000
3000
4000
5000
6000
5
10
15
20
25
Plot carbon content (g/kg)
95% CI
lpoly smooth
kernel = epanechnikov, degree = 0, bandwidth = .87, pwidth = 1.31

Figure 4.5. Rice yield as a function of plot's carbon content
Nueva Ecija, Philippines

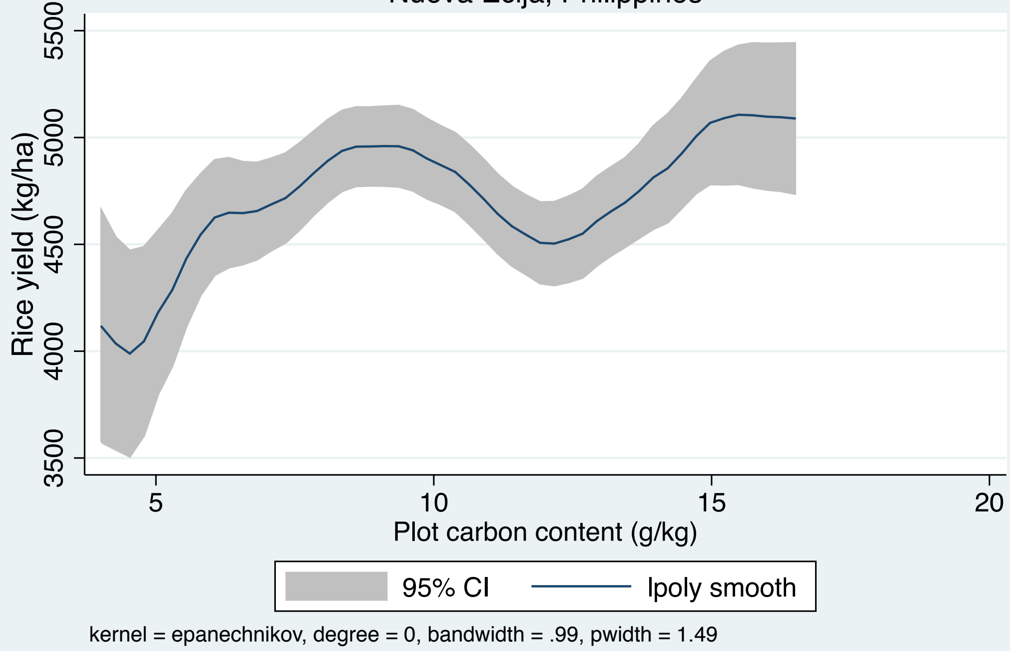


Figure 4.6. Rice yield as a function of plot's carbon content
Suphan Buri, Thailand

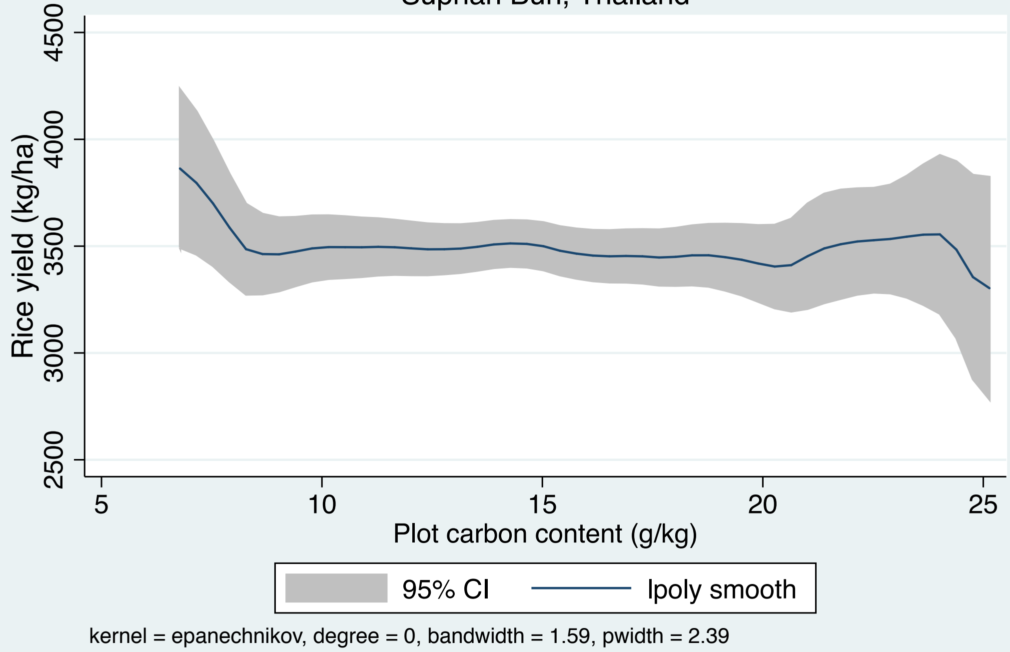
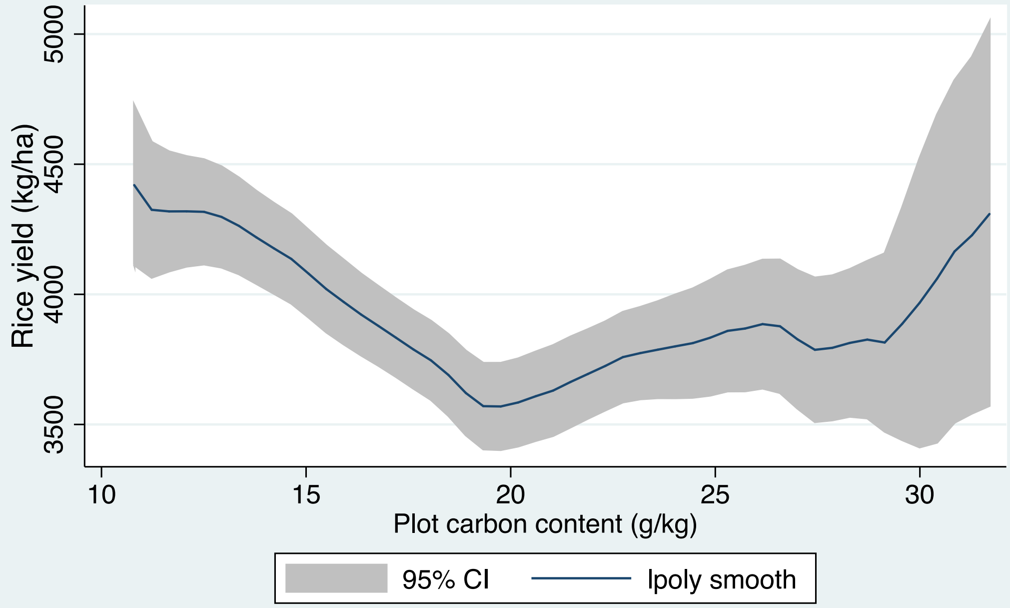
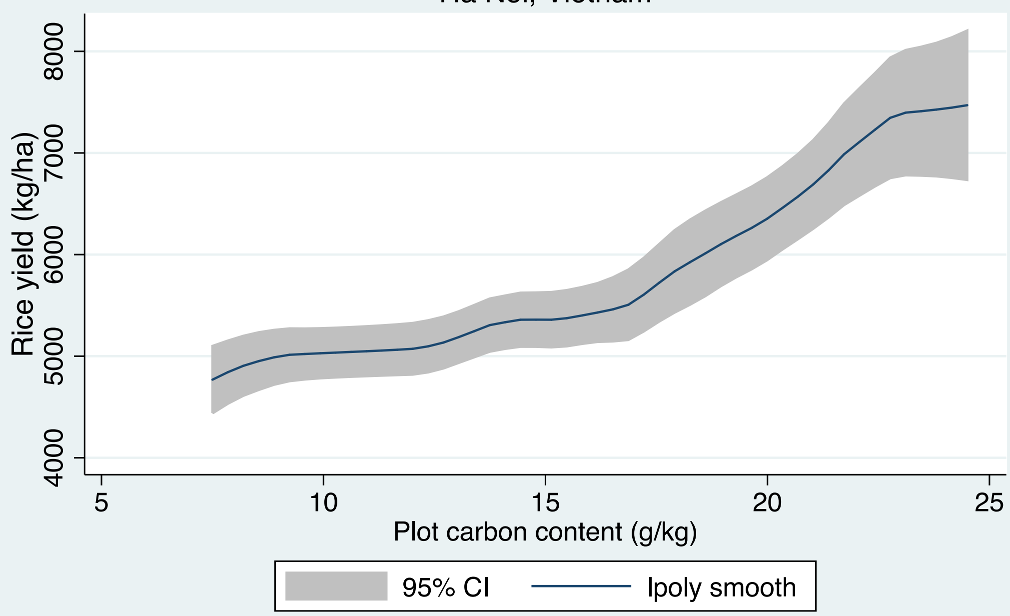


Figure 4.7. Rice yield as a function of plot's carbon content
Can Tho, Vietnam



kernel = epanechnikov, degree = 0, bandwidth = 1.81, pwidth = 2.71

Figure 4.8. Rice yield as a function of plot's carbon content
Ha Noi, Vietnam



kernel = epanechnikov, degree = 0, bandwidth = 2.08, pwidth = 3.12

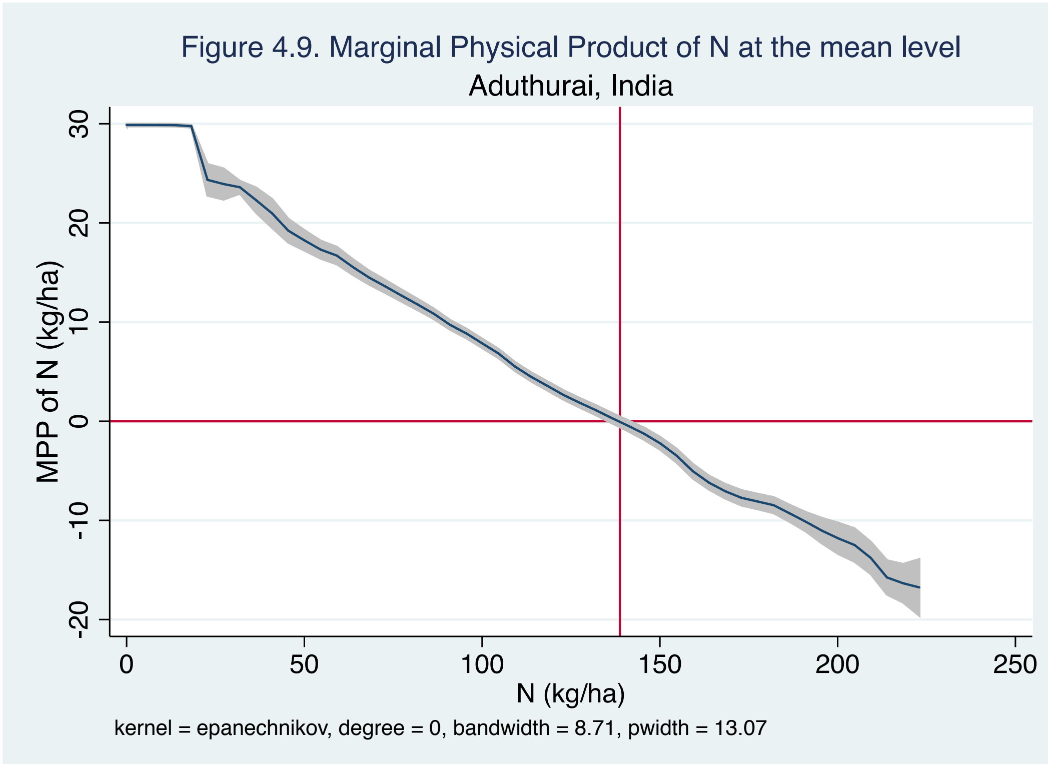


Figure 4.9. Marginal Physical Product of N at the mean level
Aduthurai, India
MPP of N (kg/ha)
30
20
10
0
-10
-20
0
50
100
150
200
250
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 8.71, pwidth = 13.07

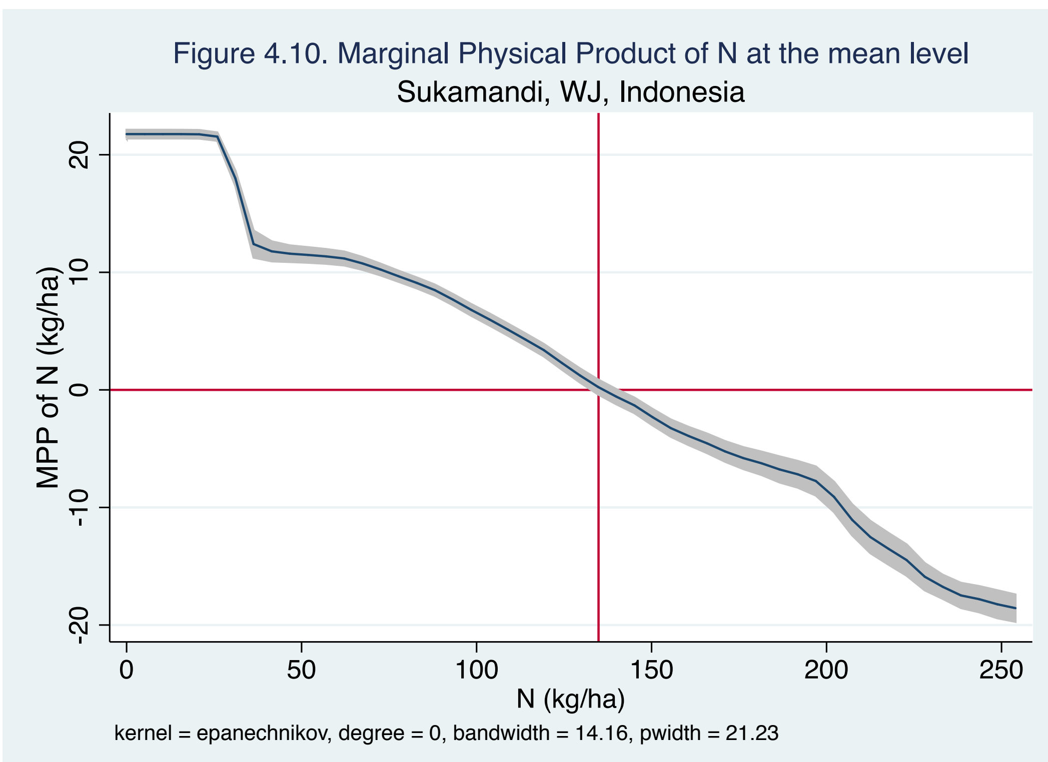


Figure 4.10. Marginal Physical Product of N at the mean level
Sukamandi, WJ, Indonesia
MPP of N (kg/ha)
20
10
0
-10
-20
0
50
100
150
200
250
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 14.16, pwidth = 21.23

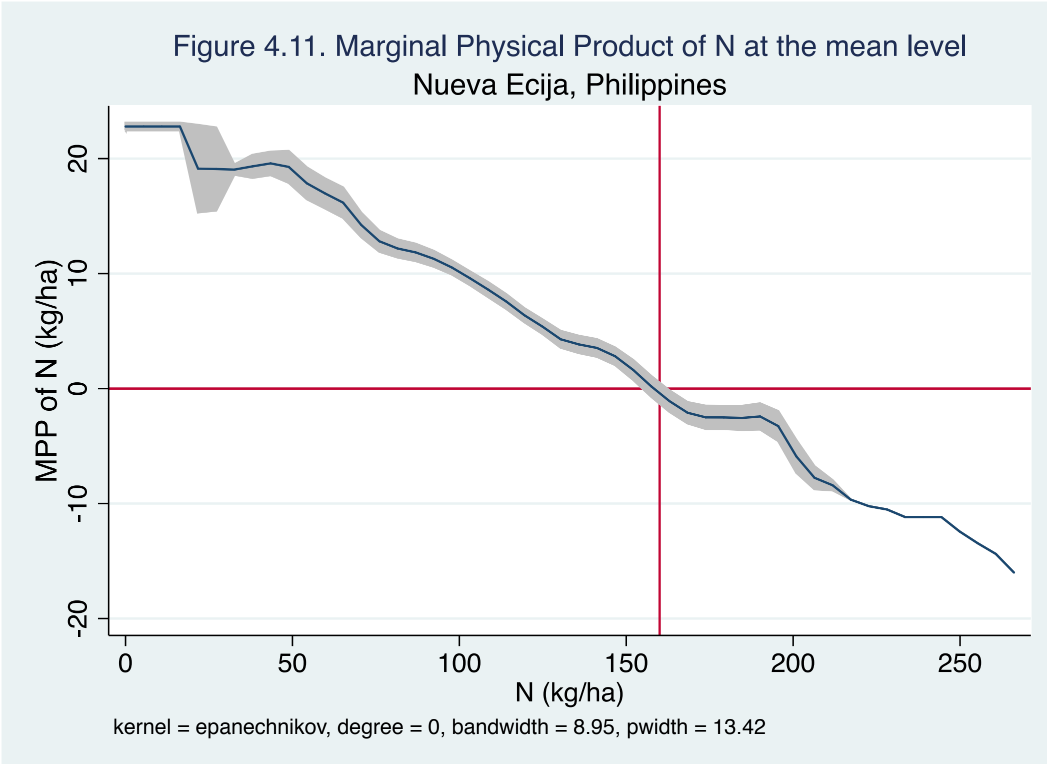


Figure 4.11. Marginal Physical Product of N at the mean level
Nueva Ecija, Philippines
MPP of N (kg/ha)
20
10
0
-10
-20
0
50
100
150
200
250
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 8.95, pwidth = 13.42

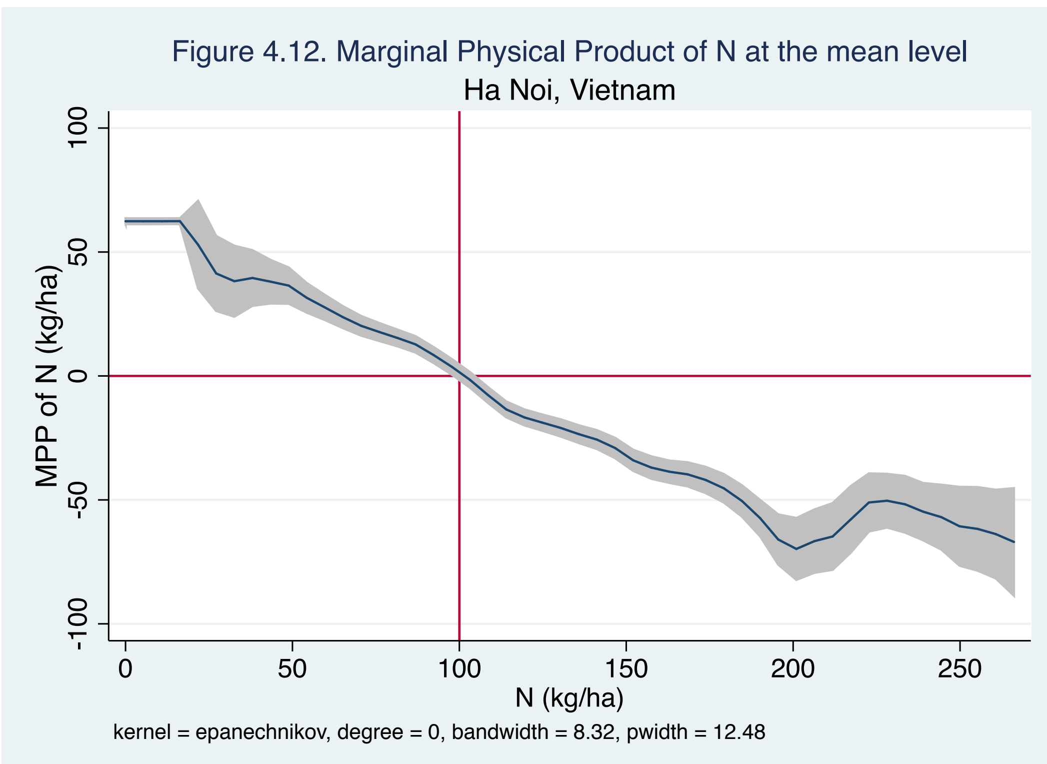


Figure 4.12. Marginal Physical Product of N at the mean level
Ha Noi, Vietnam
MPP of N (kg/ha)
100
50
0
-50
-100
0
50
100
150
200
250
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 8.32, pwidth = 12.48

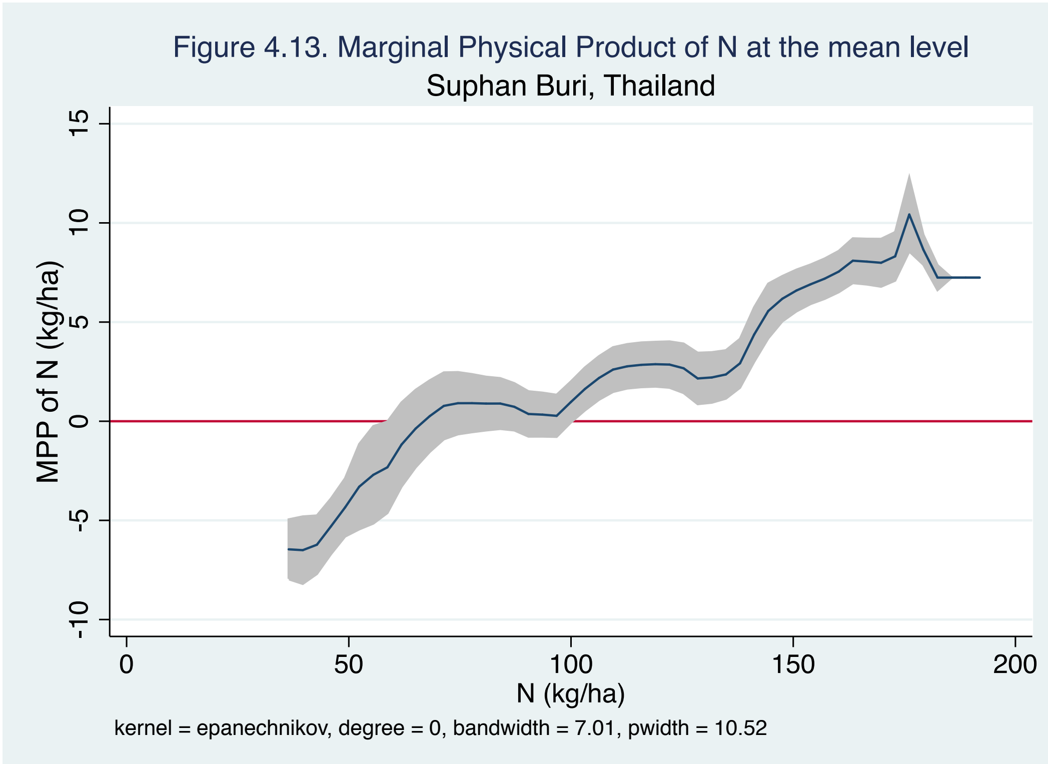


Figure 4.13. Marginal Physical Product of N at the mean level
Suphan Buri, Thailand
MPP of N (kg/ha)
-10
-5
0
5
10
15
0
50
100
150
200
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 7.01, pwidth = 10.52

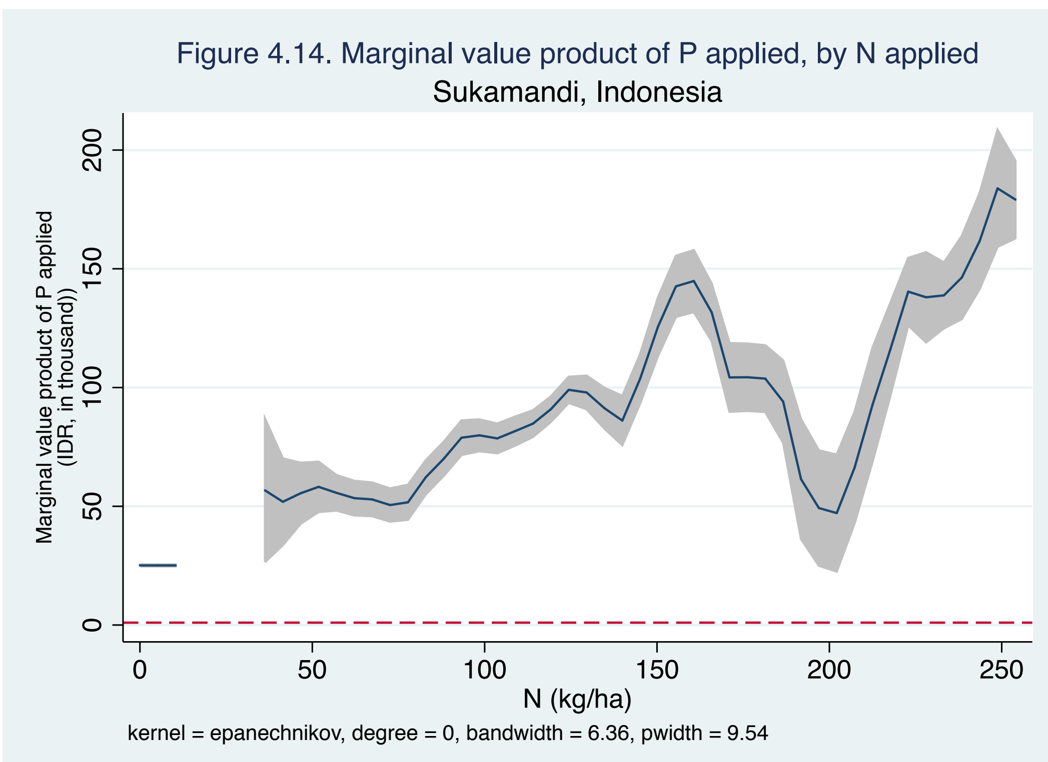


Figure 4.14. Marginal value product of P applied, by N applied
Sukamandi, Indonesia
Marginal value product of P applied (IDR, in thousand))
0
50
100
150
200
0
50
100
150
200
250
N (kg/ha)
kernel = epanechnikov, degree = 0, bandwidth = 6.36, pwidth = 9.54

Figure 4.15. Marginal value product of N applied, by P applied
Aduthurai, India

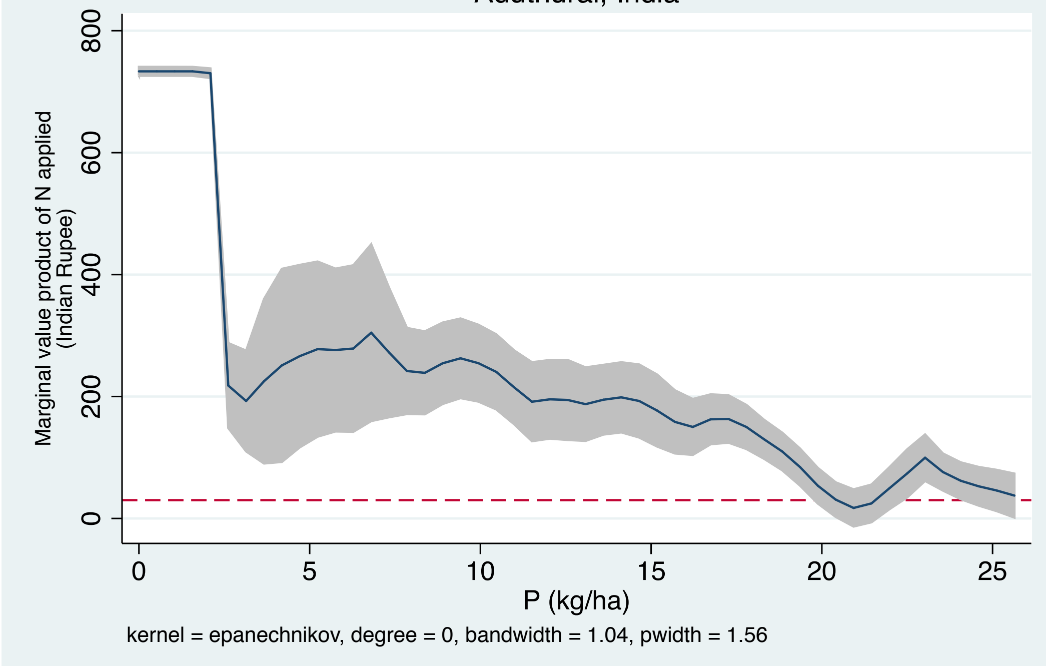


Figure 4.16. Marginal value product of N applied, by P applied
Can Tho, Vietnam

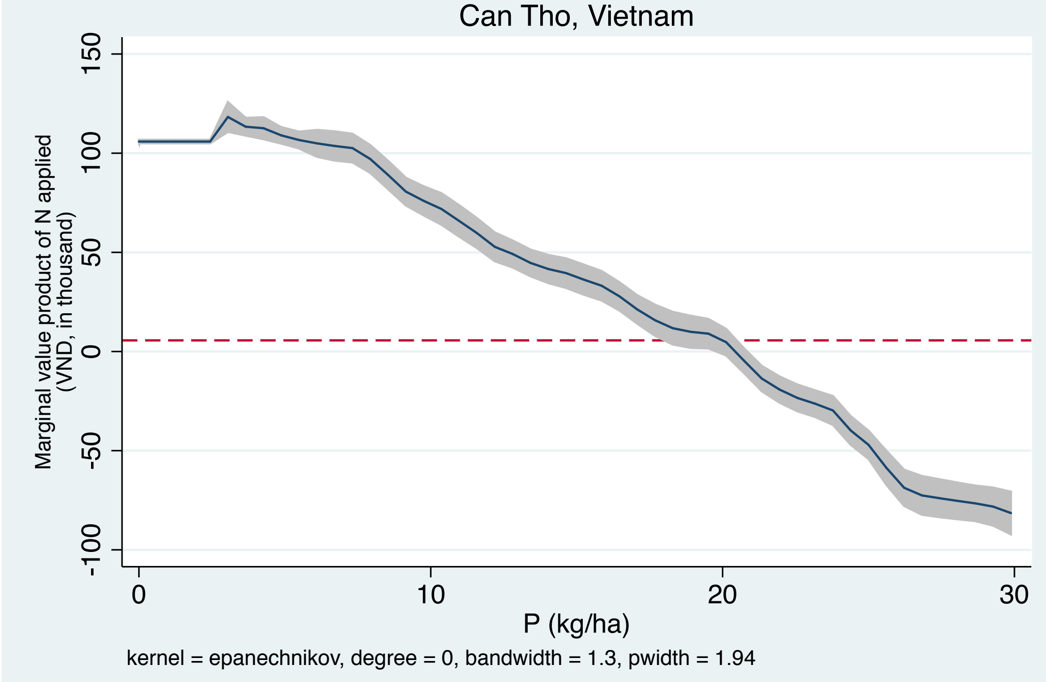


Figure 4.17. Marginal value product of N applied, by P applied
Hanoi, Vietnam

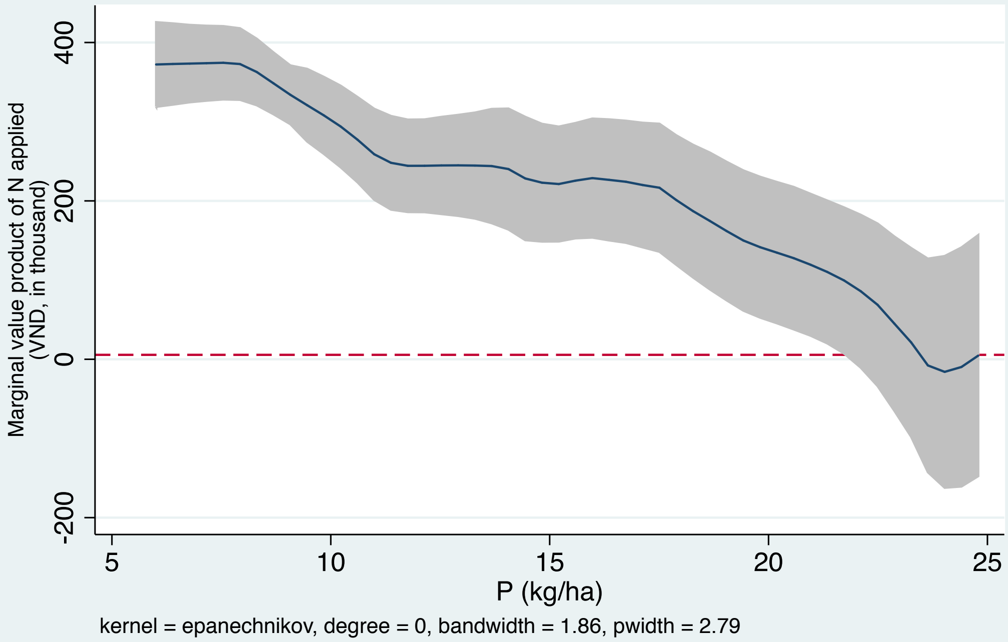


Figure 4.18. Marginal value product of N applied, by plot's carbon content
Sukamandi, Indonesia

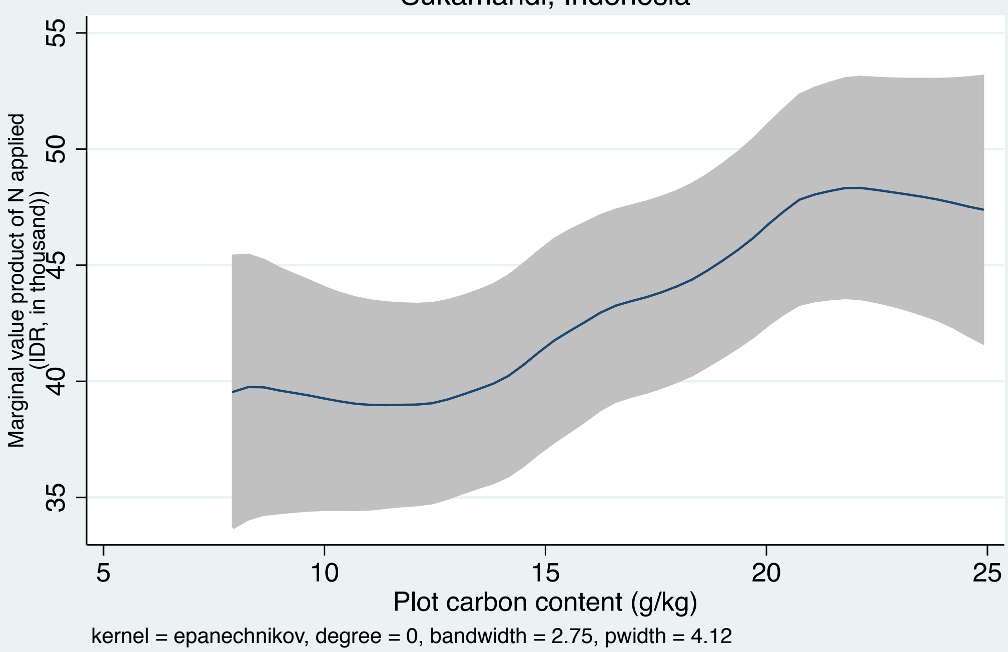


Figure 4.19. Marginal value product of N applied, by plot's carbon content
Nueva Ecija, Philippines

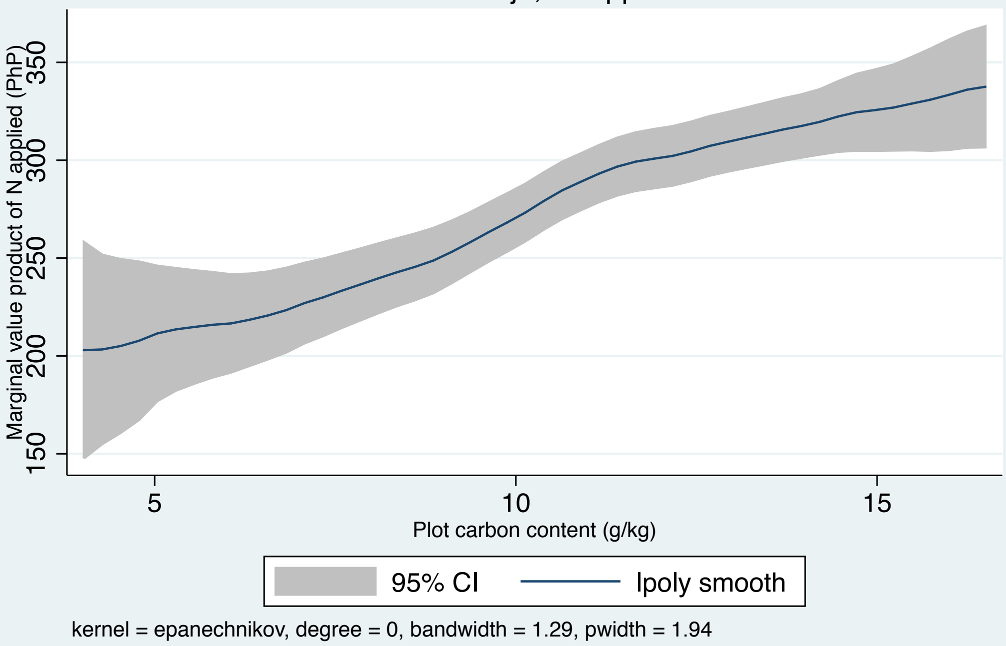
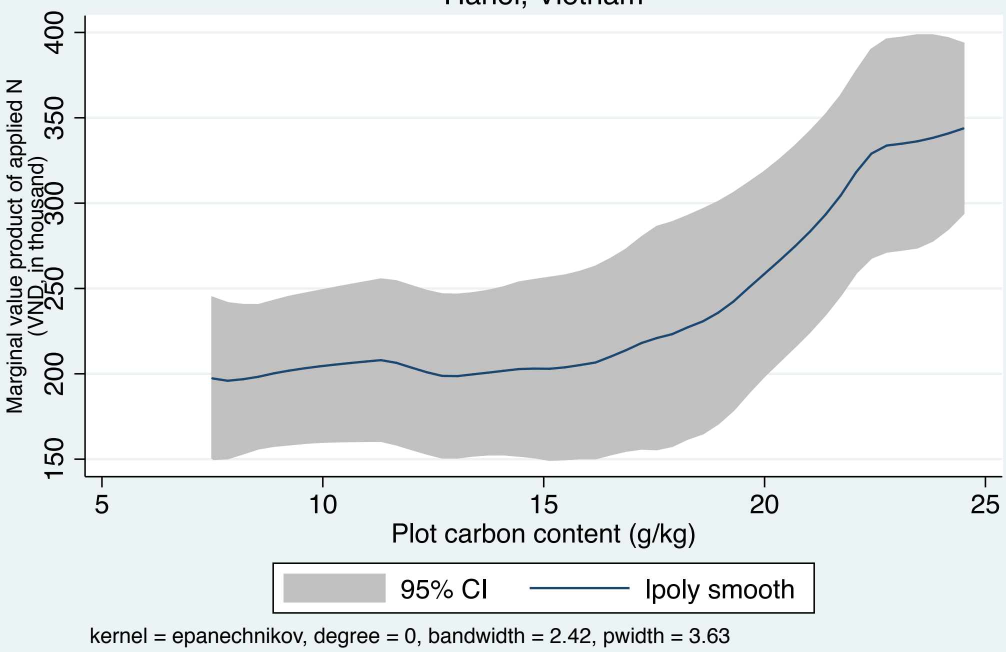


Figure 4.20. Marginal value product of N applied, by plot's carbon content
Hanoi, Vietnam



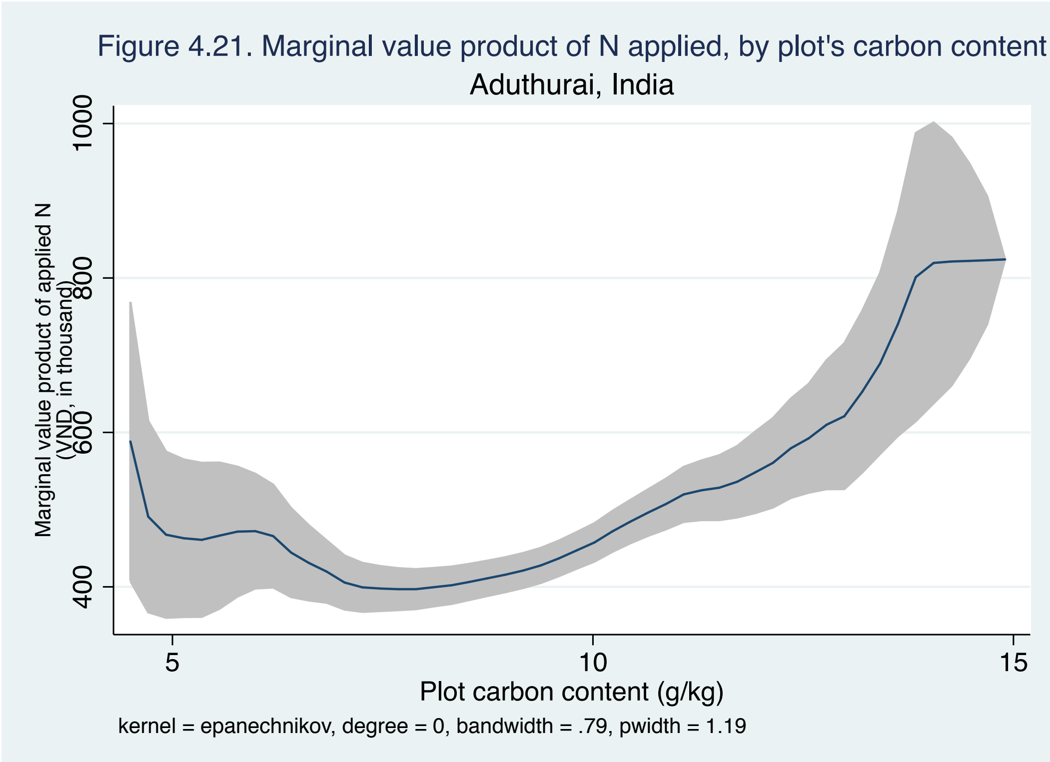


Figure 4.21. Marginal value product of N applied, by plot's carbon content
Aduthurai, India
Marginal value product of applied N
(VND, in thousand)
1000
800
600
400
5
10
15
Plot carbon content (g/kg)
kernel = epanechnikov, degree = 0, bandwidth = .79, pwidth = 1.19

Chapter 5

CONCLUSION

Globally, under- or over-application of agricultural fertilizers presents either lost agricultural production or environmental degradation. To the extent that they influence use, fertilizer recommendations can be part of the solution to these problems if made well, but may only exacerbate the problems when made poorly. In my dissertation, I evaluated two existing fertilizer recommendation algorithms that are based on yield goal approach: Stanford's 1.2 Rule for corn in the United States and site-specific nutrient management (SSNM) for rice, which uses an algorithm similar to the 1.2 Rule, in Southeast Asia.

In Chapter 2, I discussed the research from which Stanford's 1.2 Rule was developed, how it was formulated, and the reasons for its particular formulation. Fertilizer recommendations all over the world have relied heavily on an old but widely accepted rule of thumb from Stanford (1966, 1973): *apply 1.2 pounds of N fertilizer per bushel of corn expected.* With the 1.2 Rule, farmers could seemingly determine on their own, using only the yield potential of their fields and the amount of N in the soil, sufficiently accurate recommendations about their crops' fertilizer rate requirements. However, due to Stanford's lack of access to modern statistical analysis and microeconomic principles, there were numerous questionable economic and statistical procedures in the formulation of the 1.2 Rule. I used microeconomic analysis to examine the historical origins of the 1.2 Rule and show that the 1.2 Rule only makes economic sense if the crop production satisfies two restrictions: (1) it is of the von Liebig functional form, i.e. the function has a "kink" and a "plateau," and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with slope 1.2.

The limitations discussed and scrutinized in this paper suggested that sound economic theory, data from high-quality agronomic experiments, and the proper use of statistical techniques were never combined in the development of the 1.2 Rule recommendation algorithm. Although at times agronomists and agricultural economists both recognized the importance of interdisciplinary research among them, little was ever conducted. To build confidence in fertilizer recommendations, every aspect of fertilizer recommendation development, formulation, and delivery requires careful examination, including the running of field trials, the choice of response functional form when the response function is estimated, the type of data used for estimation of the economic optima, and the empirical and statistical analytical methods used. Clearly, this is an area of research in great need of interdisciplinary collaboration among agronomists and agricultural economists.

In Chapter 3, I tested if the 1.2 Rule satisfied the two restrictions above. Testing the validity of the 1.2 Rule is important to decide whether this approach to N fertilizer recommendation should be completely abandoned or followed. I used non-linear estimation techniques (i.e. non-linear least squares estimation and non-linear seemingly unrelated regression) and non-nested hypothesis framework. I also utilized the original dataset Stanford used in his analysis from Alabama, Georgia, and Mississippi, and the long-term corn experimental data from Illinois, Iowa, and Nebraska. I found no empirical evidence to support the 1.2 Rule. The linear von Liebig production function was rejected in various locations and the kinks of the von Liebig response curves for different growing conditions did not lie on a ray out of the origin with slope 1.2. The production function and the critical concentration of N can vary widely, both among states, and within states and therefore the level of optimal N also varies. Site-specificity plays a large role in determining the economic optimal fertilizer rate. It is noteworthy to revisit the fertilizer recommendation procedures that rely on the 1.2 Rule and test if it satisfies the restrictions presented in this paper. I concluded that the long-term and widespread use of Stanford's 1.2 Rule in making N fertilizer recommendation basically resulted from its long-term and widespread use.

In Chapter 4, I critically discussed and evaluated the assumptions underlying the SSNM strategy for rice, which uses an algorithm similar to the 1.2 Rule. By estimating a quadratic production function, I explored whether major nutrients are technically substitutes or complements, and whether ex ante soil conditions matter to the return on investments in inorganic fertilizer, in particular N fertilizer. I found clear evidence that interaction among major nutrients matters in making fertilizer recommendations to farmers. I also found that soil organic matter, manifested in soil carbon stocks, significantly affected the economic returns to N fertilizer inputs. Hence, the SSNM strategy should explicitly account for these factors in its algorithm.

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